ME 598: Introduction to Robotics

Lecture 8: Mobile Robot Kinematics

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Fall 2013

Date:
By:

Slides adapted from Dr. David J. Cappelleri and
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Mobile Robot Locomotion

Review

Review: Mobile Robots with Wheels

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- Wheels are the most appropriate solution for most applications
- Three wheels are sufficient to guarantee stability
- With more than three wheels an appropriate suspension is required
- Selection of wheels depends on the application

Review: The Four Basic Wheels Types

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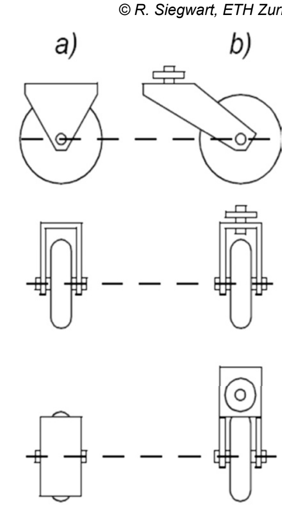
- a) Standard wheel: Two degrees of freedom; rotation around the (motorized) wheel axle and the contact point
- b) Castor wheel: Three degrees of freedom; rotation around the wheel axle, the contact point and the castor axle



Review: The Four Basic Wheels Types

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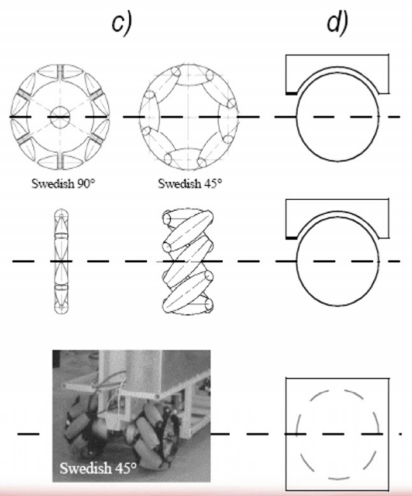
- c) Swedish wheel: Three degrees of freedom; rotation around the (motorized) wheel axle, around the rollers and around the contact point
- d) Ball or spherical wheel: Suspension technically not solved



Mobile Robot Kinematics

- Requirements for motion control

Mobile Robot Kinematics: Introduction

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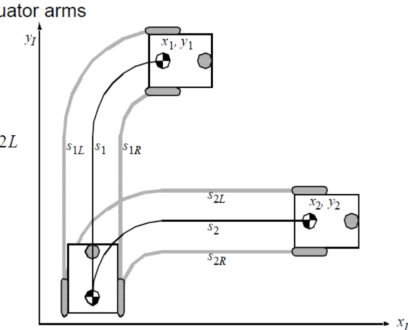
- Aim
  - Description of mechanical behavior of the robot for *design* and *control*
  - Similar to robot manipulator kinematics
- Manipulator- vs. Mobile Robot Kinematics
  - Both are concerned **with forward and inverse kinematics**
  - However, for mobile robots, encoder values don't map to unique robot poses
  - However, **mobile robots** can move unbound with respect to their environment
    - There is **no direct** (=instantaneous) **way to measure the robot's position**
    - **Position must be integrated over time**, depends on path taken
    - Leads to inaccuracies of the position (motion) estimate
  - Understanding mobile robot motion starts with **understanding wheel constraints** placed on the robot's mobility

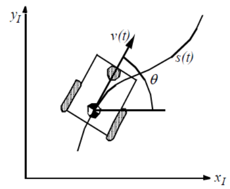
Mobile Robot Kinematics: Non-Holonomic Systems

- Non-holonomic systems
  - differential equations are not integrable to the final position.
  - the measure of the traveled distance of each wheel is not sufficient to calculate the final position of the robot. One has also to know how this movement was executed as a function of time.
  - This is in stark contrast to actuator arms

$$s_1 = s_2, s_{1R} = s_{2R}, s_{1L} = s_{2L}$$
$$x_1 \neq x_2, y_1 \neq y_2$$



Mobile Robot Kinematics: Non-Holonomic Systems



- A mobile robot is running along a trajectory *s(t)*. At every instant of the movement its velocity *v(t)* is:

$$v(t) = \frac{\partial s}{\partial t} = \frac{\partial x}{\partial t}\cos\theta + \frac{\partial y}{\partial t}\sin\theta$$

$$ds = dx\cos\theta + dy\sin\theta$$

- Function *v(t)* is said to be integrable (holonomic) if there exists a trajectory function *s(t)* that can be described by the values *x*, *y*, and θ only.

$$s = s(x, y, \theta)$$

- This is the case if

$$\frac{\partial^2 s}{\partial x \partial y} = \frac{\partial^2 s}{\partial y \partial x} \; ; \; \frac{\partial^2 s}{\partial x \partial \theta} = \frac{\partial^2 s}{\partial \theta \partial x} \; ; \; \frac{\partial^2 s}{\partial y \partial \theta} = \frac{\partial^2 s}{\partial \theta \partial y}$$

Condition for s to be integrable function

Mobile Robot Kinematics: Non-Holonomic Systems

- With $s = s(x,y,\theta)$ we get for *ds*

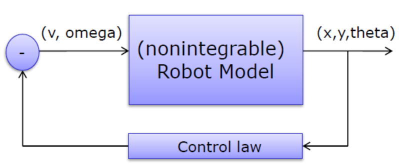
$$ds = \frac{\partial s}{\partial x}dx + \frac{\partial s}{\partial y}dy + \frac{\partial s}{\partial \theta}d\theta$$

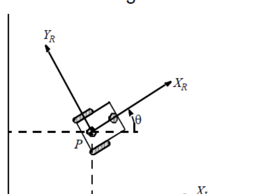
- and by comparing the equation above with $ds = dx\cos\theta + dy\sin\theta$
- we find $\frac{\partial s}{\partial x} = \cos\theta \; ; \; \frac{\partial s}{\partial y} = \sin\theta \; ; \; \frac{\partial s}{\partial \theta} = 0$
- Condition for an integrable (holonomic) function:
  - the second (-sinθ=0) and third (cosθ=0) term in the equation do not hold!

$$\frac{\partial^2 s}{\partial x \partial y} = \frac{\partial^2 s}{\partial y \partial x} \; ; \; \frac{\partial^2 s}{\partial x \partial \theta} = \frac{\partial^2 s}{\partial \theta \partial x} \; ; \; \frac{\partial^2 s}{\partial y \partial \theta} = \frac{\partial^2 s}{\partial \theta \partial y}$$

Mobile Robot Kinematics: Forward and Inverse Kinematics

- Forward kinematics:
  - Transformation from joint- to physical space
- Inverse kinematics
  - Transformation from physical- to joint space
  - Required for motion control
- Due to nonholonomic constraints in mobile robotics, we deal with **differential** (inverse) kinematics
  - Transformation between velocities instead of positions
  - Such a differential kinematic model of a robot has the following form:





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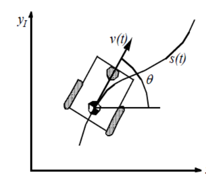
Mobile Robot Kinematics: Differential Kinematics Model

- Due to a lack of alternatives:
  - establish the robot speed $\dot{\xi} = [\dot{x} \;\; \dot{y} \;\; \dot{\theta}]^T$ as a function of the wheel speeds $\dot{\varphi}_i$, steering angles β_i, steering speeds $\dot{\beta}_i$ and the geometric parameters of the robot (*configuration coordinates*).
  - forward kinematics

$$\dot{\xi} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = f(\dot{\varphi}_1, \ldots \dot{\varphi}_n, \beta_1, \ldots \beta_m, \dot{\beta}_1, \ldots \dot{\beta}_m)$$



  - Inverse kinematics

$$[\dot{\varphi}_1 \;\; \cdots \;\; \dot{\varphi}_n \;\; \beta_1 \;\; \ldots \;\; \beta_m \;\; \dot{\beta}_1 \;\; \ldots \;\; \dot{\beta}_m]^T = f(\dot{x}, \dot{y}, \dot{\theta})$$

  - But generally not integrable into

$$\begin{bmatrix} x \\ y \\ \theta \end{bmatrix} = f(\varphi_1, \ldots \varphi_n, \beta_1, \ldots \beta_m)$$

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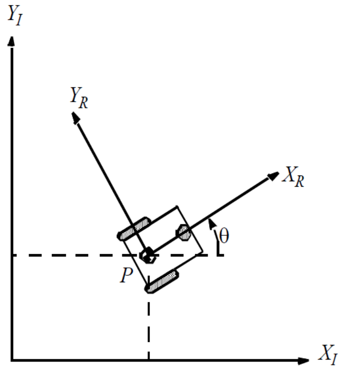
Mobile Robot Kinematics: Representing Robot Position

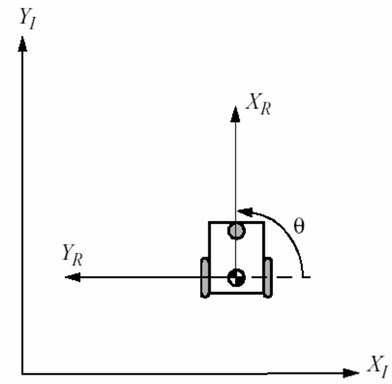
- Representing the robot within an arbitrary initial frame
  - Inertial frame: $\{X_I, Y_I\}$
  - Robot frame: $\{X_R, Y_R\}$
  - Robot pose: $\xi_I = [x \quad y \quad \theta]^T$
  - Mapping between the two frames

$$\dot{\xi}_R = R(\theta)\dot{\xi}_I = R(\theta)\cdot[\dot{x} \quad \dot{y} \quad \dot{\theta}]^T$$

$$R(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



Mobile Robot Kinematics: Example- Robot Aligned with Y_I



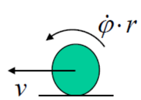
$$R(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

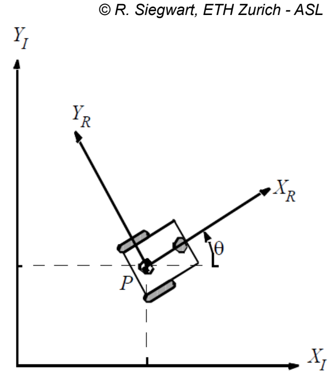
$$\dot{\xi}_R = R(\frac{\pi}{2})\dot{\xi}_I = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \dot{y} \\ -\dot{x} \\ \dot{\theta} \end{bmatrix}$$

Mobile Robot Kinematics: Wheel Kinematic Constraints

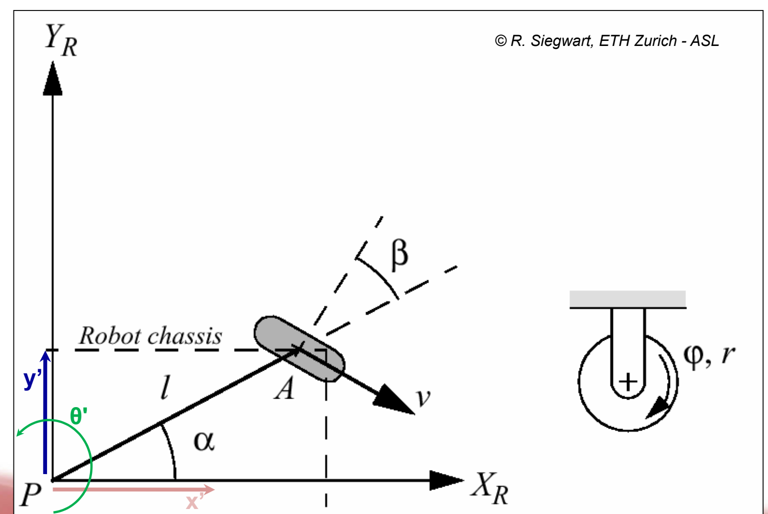
Assumptions:

- Movement on a horizontal plane
- Point contact of the wheels
- Wheels not deformable
- Pure rolling
  - $v_c = 0$ at contact point
- No slipping, skidding or sliding
- No friction for rotation around contact point
- Steering axes orthogonal to the surface
- Wheels connected by rigid frame (chassis)

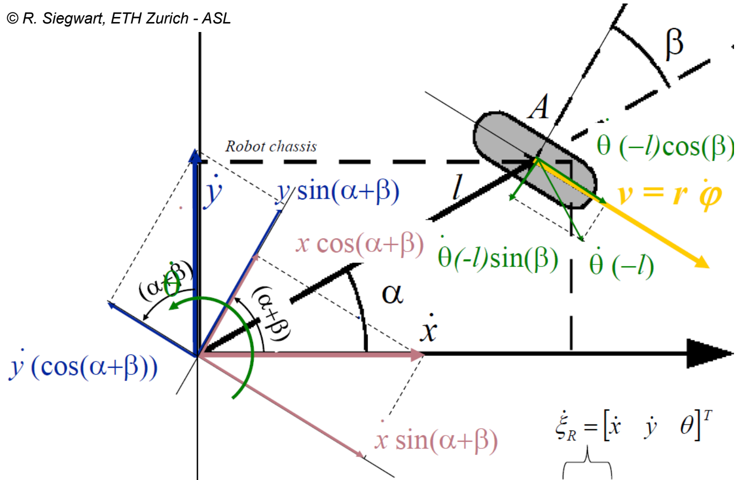




Mobile Robot Kinematics: Wheel Kinematic Constraints- Fixed Standard Wheel



Mobile Robot Kinematics: Wheel Kinematic Constraints- Fixed Standard Wheel



$$\dot{\xi}_R = [\dot{x} \quad \dot{y} \quad \dot{\theta}]^T$$

$$\left[\sin(\alpha+\beta) \;\; -\cos(\alpha+\beta) \;\; (-l)\cos\beta\right] R(\theta)\dot{\xi}_I - r\dot{\varphi} = 0$$

$$\left[\cos(\alpha+\beta) \;\; \sin(\alpha+\beta) \;\; l\sin\beta\right] R(\theta)\dot{\xi}_I = 0$$

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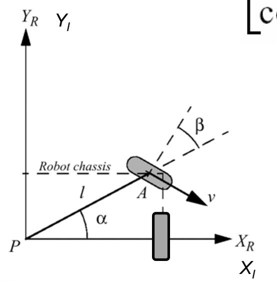
Mobile Robot Kinematics: Constraints- Fixed Standard Wheel Example

- Suppose that the wheel A is in position such that $\alpha = 0$ and $\beta = 0$
- This would place the contact point of the wheel on X_I with the plane of the wheel oriented parallel to Y_I. If $\theta = 0$, then the sliding constraint reduces to:

$$\left[\cos(\alpha+\beta) \;\; \sin(\alpha+\beta) \;\; l\sin\beta\right] R(\theta)\dot{\xi}_I = 0$$

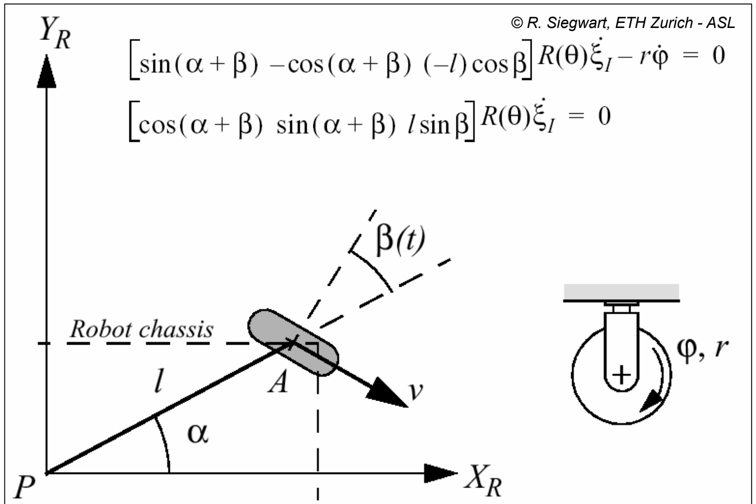


$$\begin{bmatrix}1 & 0 & 0\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}\dot{x}\\\dot{y}\\\dot{\theta}\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\end{bmatrix}\begin{bmatrix}\dot{x}\\\dot{y}\\\dot{\theta}\end{bmatrix} = 0$$

Mobile Robot Kinematics: Wheel Kinematic Constraints- Steered Standard Wheel

$$\left[\sin(\alpha+\beta) \;\; -\cos(\alpha+\beta) \;\; (-l)\cos\beta\right] R(\theta)\dot{\xi}_I - r\dot{\varphi} = 0$$

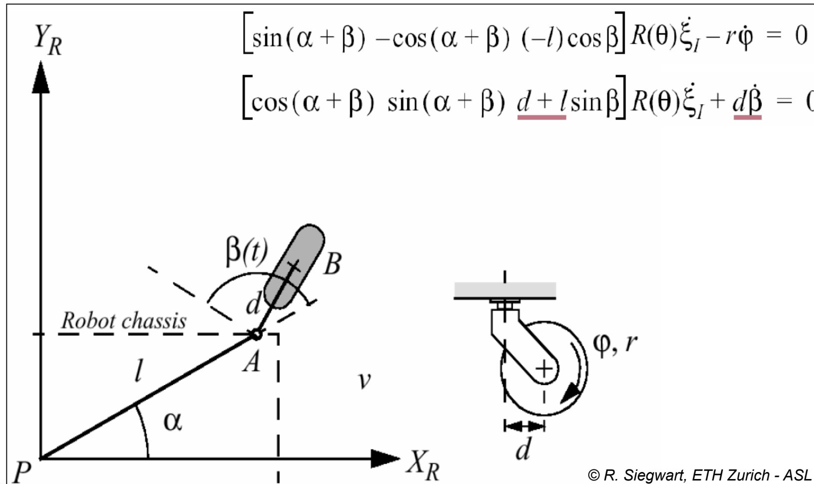
$$\left[\cos(\alpha+\beta) \;\; \sin(\alpha+\beta) \;\; l\sin\beta\right] R(\theta)\dot{\xi}_I = 0$$



Mobile Robot Kinematics: Wheel Kinematic Constraints- Castor Wheel

$$\left[\sin(\alpha+\beta) \;\; -\cos(\alpha+\beta) \;\; (-l)\cos\beta\right] R(\theta)\dot{\xi}_I - r\dot{\varphi} = 0$$

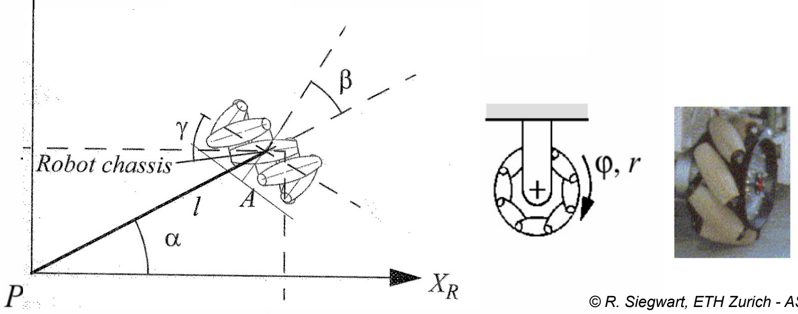
$$\left[\cos(\alpha+\beta) \;\; \sin(\alpha+\beta) \;\; \underline{d} + l\sin\beta\right] R(\theta)\dot{\xi}_I + \underline{d\dot{\beta}} = 0$$



Mobile Robot Kinematics: Wheel Kinematic Constraints- Swedish Wheel

$$\begin{bmatrix}\sin(\alpha+\beta+\gamma) & -\cos(\alpha+\beta+\gamma) & (-l)\cos(\beta+\gamma)\end{bmatrix} R(\theta)\dot{\xi}_I - r\dot{\varphi}\cos\gamma = 0$$

$$\begin{bmatrix}\cos(\alpha+\beta+\gamma) & \sin(\alpha+\beta+\gamma) & l\sin(\beta+\gamma)\end{bmatrix} R(\theta)\dot{\xi}_I - r\dot{\varphi}\sin\gamma - r_{sw}\dot{\varphi}_{sw} = 0$$



Mobile Robot Kinematics: Kinematic Constraints- Complete Robot

- Given a robot with *M* wheels
  - each wheel imposes zero or more constraints on the robot motion
  - only fixed and steerable standard wheels impose constraints
- What is the maneuverability of a robot considering a combination of different wheels?
- Suppose we have a total of $N=N_f+N_s$ standard wheels
  - We can develop the equations for the constraints in matrix forms:
  - Rolling

$$J_1(\beta_s)R(\theta)\dot{\xi}_I + J_2\dot{\varphi} = 0 \qquad \varphi(t) = \begin{bmatrix}\varphi_f(t)\\ \varphi_s(t)\end{bmatrix}_{(N_f+N_s)\times 1} \qquad J_1(\beta_s) = \begin{bmatrix}J_{1f}\\ J_{1s}(\beta_s)\end{bmatrix}_{(N_f+N_s)\times 3} \qquad J_2 = diag(r_1 \cdots r_N)$$

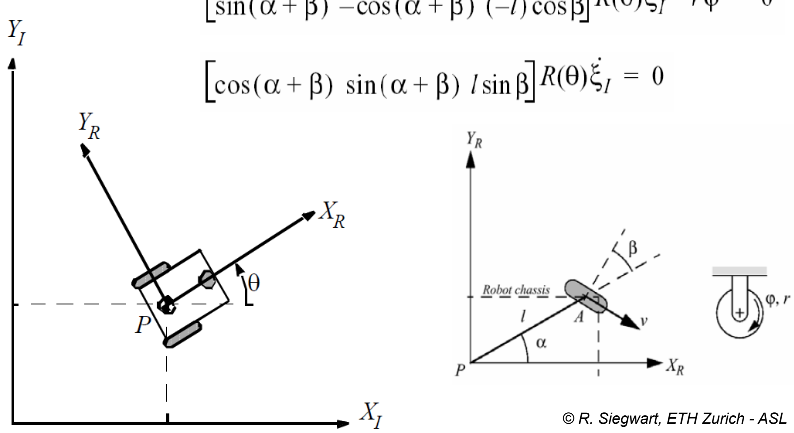
  - Lateral movement

$$C_1(\beta_s)R(\theta)\dot{\xi}_I = 0 \qquad C_1(\beta_s) = \begin{bmatrix}C_{1f}\\ C_{1s}(\beta_s)\end{bmatrix}_{(N_f+N_s)\times 3}$$

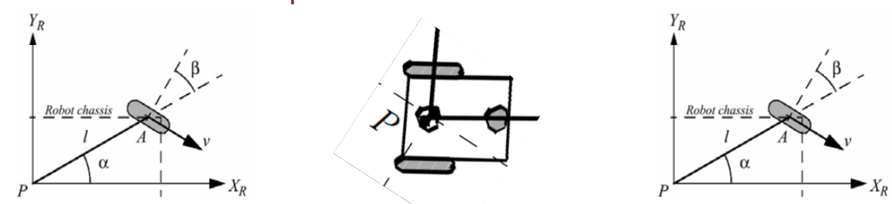
Mobile Robot Kinematics: Example- Differential Drive Robot

$$\begin{bmatrix}\sin(\alpha+\beta) & -\cos(\alpha+\beta) & (-l)\cos\beta\end{bmatrix} R(\theta)\dot{\xi}_I - r\dot{\varphi} = 0$$

$$\begin{bmatrix}\cos(\alpha+\beta) & \sin(\alpha+\beta) & l\sin\beta\end{bmatrix} R(\theta)\dot{\xi}_I = 0$$

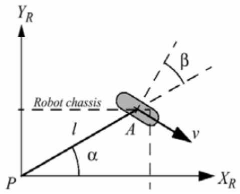


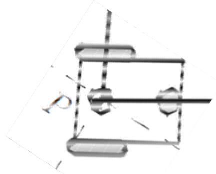
Mobile Robot Kinematics: Example- Differential Drive Robot

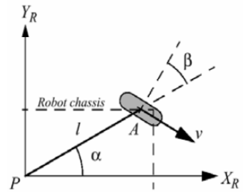


$$\begin{bmatrix}\sin(\alpha+\beta) & -\cos(\alpha+\beta) & (-l)\cos\beta\end{bmatrix} R(\theta)\dot{\xi}_I - r\dot{\varphi} = 0$$

Mobile Robot Kinematics: Example- Differential Drive Robot







$$\left[\cos(\alpha+\beta)\ \ \sin(\alpha+\beta)\ \ l\sin\beta\right]R(\theta)\dot{\xi}_I = 0$$

Mobile Robot Kinematics: Mobile Robot Maneuverability

- The maneuverability of a mobile robot is the combination
  - of the mobility available based on the sliding constraints
  - plus additional freedom contributed by the steering
- Three wheels is sufficient for static stability
  - additional wheels need to be synchronized
  - this is also the case for some arrangements with three wheels
- It can be derived using the equation seen before
  - Degree of mobility δ_m
  - Degree of steerability δ_s
  - Robots maneuverability $\delta_M = \delta_m + \delta_s$

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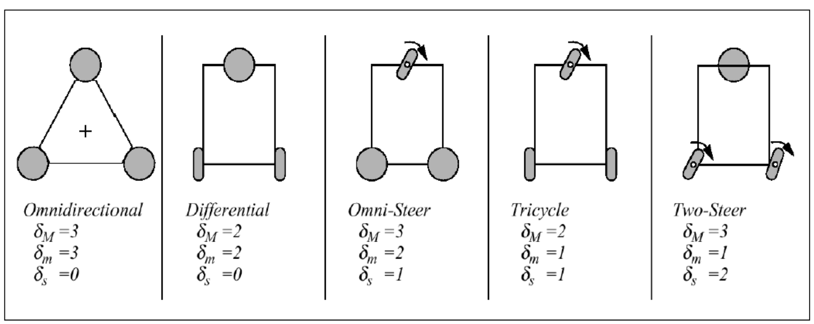
Mobile Robot Kinematics: Mobile Robot Maneuverability



Degree of mobility δ_m
Degree of steerability δ_s
Robots maneuverability $\delta_M = \delta_m + \delta_s$

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Mobile Robot Kinematics: Mobile Robot Workspace- DoF

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- The Degree of Freedom (DOF) is the robot's ability to achieve various poses.
- But what is the degree of vehicle's freedom in its environment?
- Workspace
  - how the vehicle is able to move between different configuration in its workspace?
- The robot's independently achievable velocities
  - = *differentiable degrees of freedom (DDOF)* = δ_m
  - Bicycle: $\delta_M = \delta_m + \delta_s = 1 + 1$ DDOF = 1; DOF=3
  - Omni Drive: $\delta_M = \delta_m + \delta_s = 3 + 0$ DDOF=3; DOF=3

Mobile Robot Kinematics: Degrees of Freedom, Holonomy

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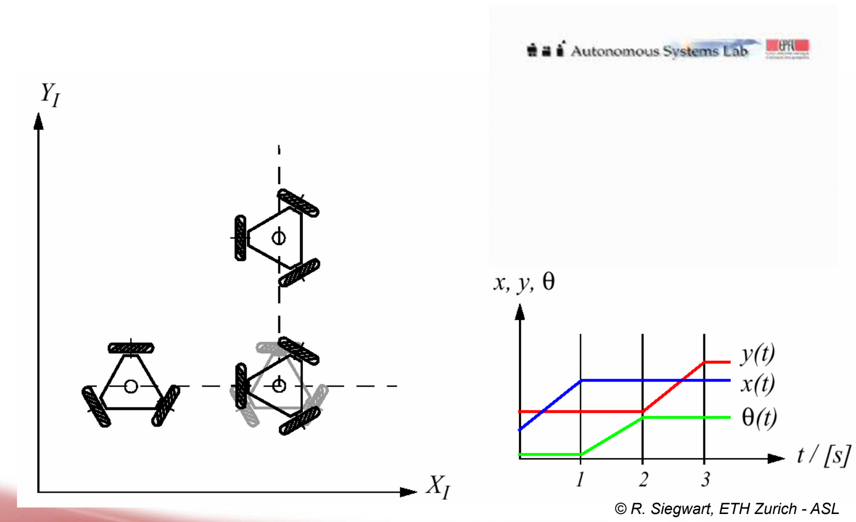
- DOF *degrees of freedom*:
  - Robots ability to achieve various poses
- DDOF *differentiable degrees of freedom*:
  - Robots ability to achieve various path

How many DOF can be controlled by just changing wheel velocities

$$DDOF \leq \delta_m \leq DOF$$

- Holonomic Robots
  - A holonomic kinematic constraint can be expressed a an explicit function of position variables only
  - A non-holonomic constraint requires a different relationship, such as the derivative of a position variable
  - *Fixed and steered standard wheels impose non-holonomic constraints*

Mobile Robot Kinematics: Path / Trajectory Considerations- Omnidirectional Drive



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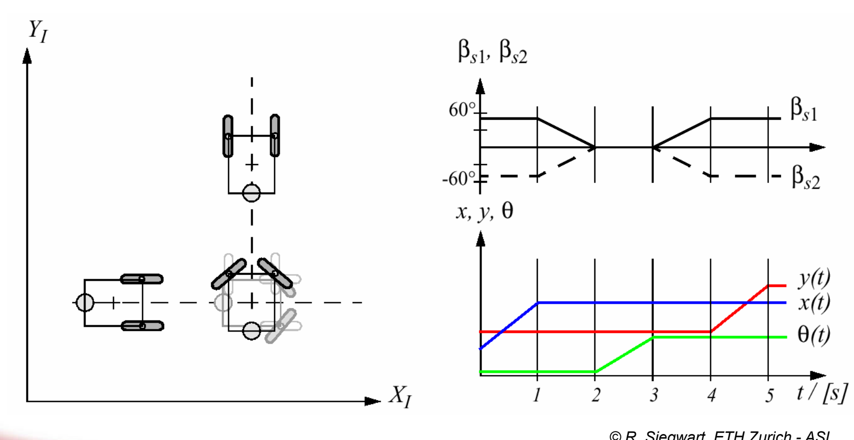
Mobile Robot Kinematics: Path / Trajectory Considerations- Two-Steer Drive



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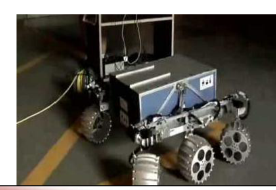
Mobile Robot Kinematics: Beyond Basic Kinematics…

- At higher speeds, and in difficult terrain, dynamics become important

- For other vehicles, the no-sliding constraints, and simple kinematics presented in this lecture do not hold



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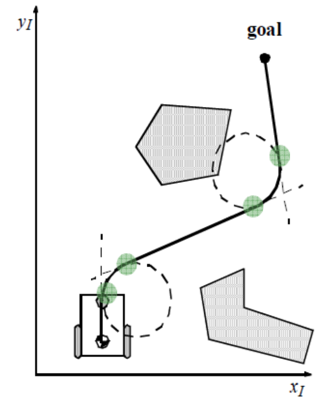
Mobile Robot Kinematics: Wheeled Mobile Robot Motion Control- Overview

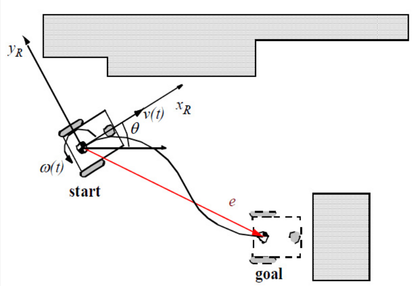
- The objective of a kinematic controller is to follow a trajectory described by its position and/or velocity profiles as function of time.
- Motion control is not straight forward because mobile robots are typically non-holonomic and MIMO systems.
- Most controllers (including the one presented here) are not considering the dynamics of the system

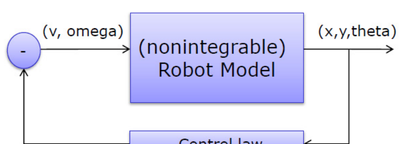
Mobile Robot Kinematics: Motion Control- Open-Loop

- trajectory (path) divided in motion segments of clearly defined shape:
  - straight lines and segments of a circle.
- control problem:
  - pre-compute a smooth trajectory based on line and circle segments
- Disadvantages:
  - It is not at all an easy task to pre-compute a feasible trajectory
  - limitations and constraints of the robots velocities and accelerations
  - does not adapt or correct the trajectory if dynamical changes of the environment occur.
  - The resulting trajectories are usually not smooth



Mobile Robot Kinematics: Motion Control- Feedback Control





- Find a control matrix K, if exists

$$K = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \end{bmatrix}$$

with $k_{ij}=k(t,e)$

- such that the control of $v(t)$ and $\omega(t)$

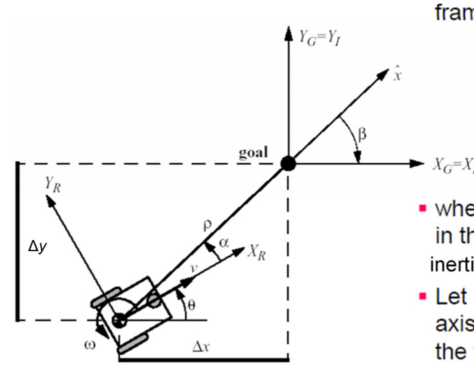
$$\begin{bmatrix} v(t) \\ \omega(t) \end{bmatrix} = K \cdot e = K \cdot {}^R\begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$$

- drives the error e to zero

$$\lim_{t\to\infty} e(t) = 0$$

- MIMO state feedback control

Mobile Robot Kinematics: Motion Control- Kinematic Model



- The kinematics of a differential drive mobile robot described in the inertial frame $\{x_I, y_I, \theta\}$ is given by,

$${}^I\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} cos\theta & 0 \\ sin\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}$$

- where $\dot{x}$ and $\dot{y}$ are the linear velocities in the direction of the x_I and y_I of the inertial frame.
- Let α denote the angle between the x_R axis of the robots reference frame and the vector connecting the center of the axle of the wheels with the final position.

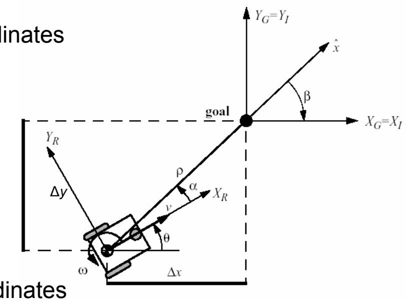
Mobile Robot Kinematics: Kinematic Model: Coordinate Transformation

- Coordinate transformation into polar coordinates with its origin at goal position:

$$\rho = \sqrt{\Delta x^2 + \Delta y^2}$$

$$\alpha = -\theta + \operatorname{atan2}(\Delta y, \Delta x)$$

$$\beta = -\theta - \alpha$$



- System description, in the new polar coordinates

$$\begin{bmatrix}\dot\rho\\ \dot\alpha\\ \dot\beta\end{bmatrix} = \begin{bmatrix}-\cos\alpha & 0\\ \frac{\sin\alpha}{\rho} & -1\\ -\frac{\sin\alpha}{\rho} & 0\end{bmatrix}\begin{bmatrix}v\\ \omega\end{bmatrix}$$

for $\alpha \in I_1 = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$$\begin{bmatrix}\dot\rho\\ \dot\alpha\\ \dot\beta\end{bmatrix} = \begin{bmatrix}\cos\alpha & 0\\ -\frac{\sin\alpha}{\rho} & 1\\ \frac{\sin\alpha}{\rho} & 0\end{bmatrix}\begin{bmatrix}v\\ \omega\end{bmatrix}$$

for $\alpha \in I_2 = (-\pi, -\pi/2] \cup (\pi/2, \pi]$

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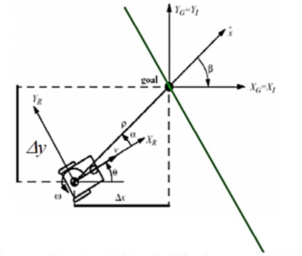
Mobile Robot Kinematics: Kinematic Model- Coordinate Transformation Remarks

- The coordinates transformation is not defined at x = y = 0; as in such a point the determinant of the Jacobian matrix of the transformation is not defined, i.e. it is unbounded
- For $\alpha \in I_1$ the forward direction of the robot points toward the goal, for $\alpha \in I_2$ it is the backward direction.

$$\alpha \in I_1 = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right]$$



- By properly defining the forward direction of the robot at its initial configuration, it is always possible to have $\alpha \in I_1$ at t=0. However this does not mean that a remains in I_1 for all time t.

Mobile Robot Kinematics: Kinematic Position Control- Control Law

- It can be shown, that with

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$$v = k_\rho \rho \qquad \omega = k_\alpha \alpha + k_\beta \beta$$

the feedback controlled system

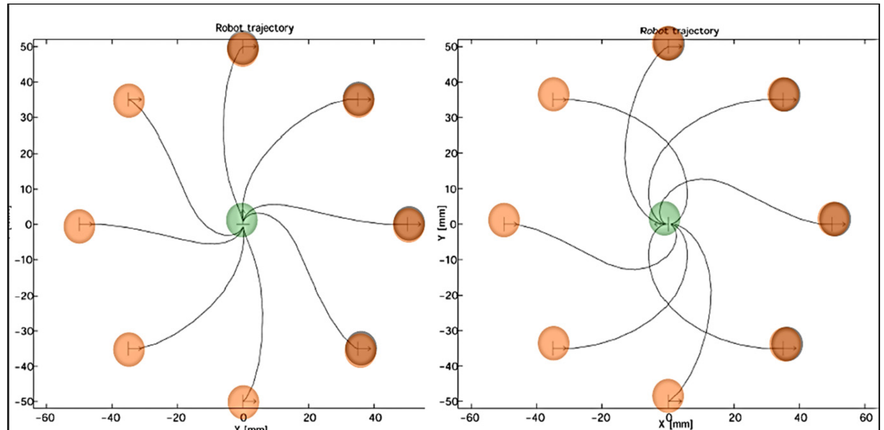
$$\begin{bmatrix}\dot\rho\\ \dot\alpha\\ \dot\beta\end{bmatrix} = \begin{bmatrix}-k_\rho \rho \cos\alpha\\ k_\rho \sin\alpha - k_\alpha \alpha - k_\beta \beta\\ -k_\rho \sin\alpha\end{bmatrix}$$

will drive the robot to $(\rho, \alpha, \beta) = (0,0,0)$

- The control signal v has always constant sign,
  - the direction of movement is kept positive or negative during movement
  - parking maneuver is performed always in the most natural way and without ever inverting its motion.

Mobile Robot Kinematics: Kinematic Position Control- Resulting Path

The goal is in the center and the initial position on the circle



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$$\mathrm{k} = (k_\rho, k_\alpha, k_\beta) = (3, 8, -1.5)$$