ME 598: Introduction to Robotics

Lecture 3: Velocity Kinematics- The Jacobian

Stevens Institute of Technology
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Date:
By:

Slides adapted from Dr. David J. Cappelleri
Some slides courtesy of Jonathan Fiene, University of Pennsylvania

STEVENS INSTITUTE of TECHNOLOGY THE INNOVATION UNIVERSITY

Review: Forward Kinematics

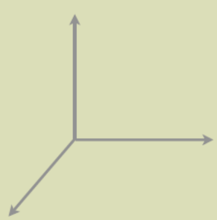
Review: Denavit-Hartenberg Convention

The **Denavit-Hartenberg convention** defines four parameters and some rules to help characterize arbitrary kinematic chains

start by attaching a frame to each link:

the joint variable for link i+1 acts along/around z_i

the axis x_i is perpendicular to, and intersects z_{i-1}



the following conventions make this process easier:

if z_{i-1} is parallel to z_i	orient x_i along normal with z_{i-1}
if z_{i-1} intersects z_i	orient x_i normal to the plane formed by z_{i-1} and z_i
if z_{i-1} is not coplanar with z_i	orient x_i along normal with z_{i-1}

Denavit & Hartenberg, "A kinematic notation for lower-pair mechanisms based on matrices," ASME Journal of Applied Mechanics, June 1955

Review: Denavit-Hartenberg Convention

The **Denavit-Hartenberg convention** defines four parameters and some rules to help characterize arbitrary kinematic chains

a_i **Link Length**	the distance perpendicular to z_i and z_{i-1}, measured along x_i
α_i **Link Twist**	the angle between z_{i-1} and z_i, measured in the plane normal to x_i (right-hand rule around x_i)
d_i **Link Offset**	the distance along z_{i-1} from o_{i-1} to the intersection with x_i
θ_i **Joint Angle**	the angle between x_{i-1} and x_i, measured in the plane normal to z_{i-1} (right-hand rule around z_{i-1})

Denavit & Hartenberg, "A kinematic notation for lower-pair mechanisms based on matrices," ASME Journal of Applied Mechanics, June 1955

Review: Denavit-Hartenberg Transform

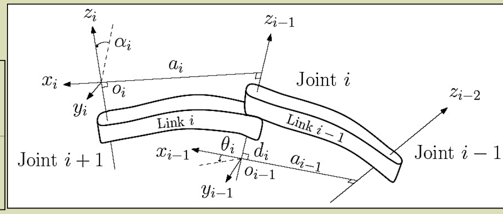
The **Denavit-Hartenberg transform** results from successive rotations and translations via the four DH parameters

The transform from i-1 to i:

$$A_i = \text{Rot}_{z,\theta_i}\ \text{Trans}_{z,d_i}\ \text{Trans}_{x,a_i}\ \text{Rot}_{x,\alpha_i}$$

$$= \begin{bmatrix} c_{\theta_i} & -s_{\theta_i}c_{\alpha_i} & s_{\theta_i}s_{\alpha_i} & a_i c_{\theta_i} \\ s_{\theta_i} & c_{\theta_i}c_{\alpha_i} & -c_{\theta_i}s_{\alpha_i} & a_i s_{\theta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

a_i **Link Length**	d_i **Link Offset**
α_i **Link Twist**	θ_i **Joint Angle**



Review: Inverse Kinematics

Review: Algebraic Decomposition

given the forward transform matrix for a manipulator

$$\mathbf{T}_n^0 = \begin{bmatrix} [\mathbf{R}_n^0(\mathbf{q})]_{3\times3} & [\mathbf{d}_n^0(\mathbf{q})]_{3\times1} \\ [\mathbf{0}]_{1\times3} & 1 \end{bmatrix}$$

solve the system of 3 equations from the displacement vector

$$\begin{aligned} d_x &= [\mathbf{d}_n^0(\mathbf{q})]_1 \\ d_y &= [\mathbf{d}_n^0(\mathbf{q})]_2 \\ d_z &= [\mathbf{d}_n^0(\mathbf{q})]_3 \end{aligned}$$

to find the joint variables in terms of the end-effector position

$$\mathbf{q} = \begin{bmatrix} q_1(d_x, d_y, d_z) \\ q_2(d_x, d_y, d_z) \\ \vdots \\ q_n(d_x, d_y, d_z) \end{bmatrix}$$

Review: Geometric Analysis

For most simple manipulators, it is often easier to use geometry to solve for closed-form solutions to the inverse kinematics

solve for each joint variable q_i by projecting the manipulator onto the x_{i-1}, y_{i-1} plane

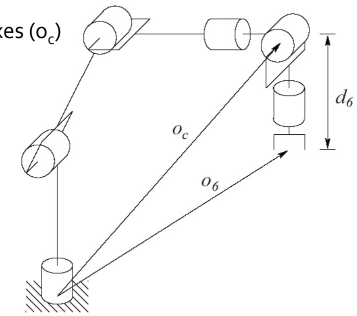
closed-form inverse kinematic solutions are not always possible, and if it is solvable, there are often multiple solutions

Review: Kinematic Decoupling

Inverse kinematics =
inverse position kinematics +
inverse orientation kinematics

Two sub problems:

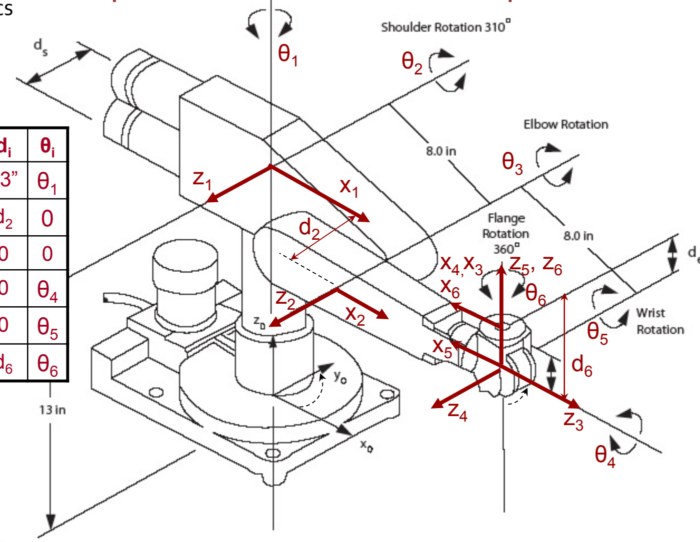
- Find position of the intersection of the wrist axes (o_c)
- Find orientation of the wrist



Review: Example P3.10- PUMA 360 Manipulator

Forward Kinematics

Link	a_i	α_i	d_i	θ_i
1	0	90	13"	θ_1
2	8"	0	d_2	0
3	8"	90	0	0
4	0	-90	0	θ_4
5	0	90	0	θ_5
6	0	0	d_6	θ_6

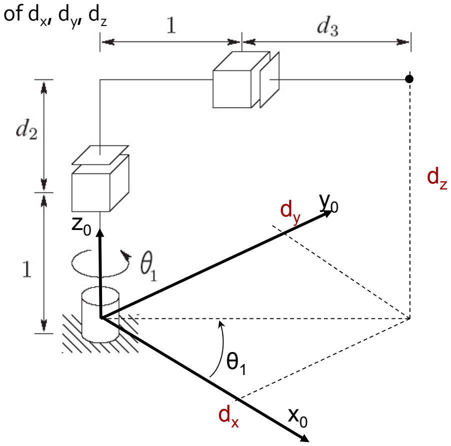


Review: Example P3.13- Cylindrical Manipulator

Inverse Kinematics

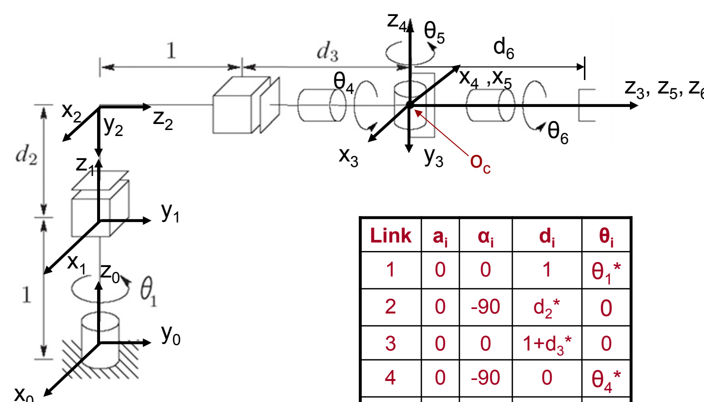
Given: $d = [d_x, d_y, d_z]^T$

Find: θ_1, d_2, and d_3 as functions of d_x, d_y, d_z



solve for each joint variable q_i by projecting the manipulator onto the x_{i-1}, y_{i-1} plane

Review: Example P3.15- Cylindrical Manipulator + Spherical Wrist



Link	a_i	α_i	d_i	θ_i
1	0	0	1	θ_1*
2	0	-90	d_2*	0
3	0	0	1+d_3*	0
4	0	-90	0	θ_4*
5	0	90	0	θ_5*
6	0	0	d_6	θ_6*

Velocity Kinematics: The Jacobian

The Jacobian: Differential Motion

The Instantaneous Position Jacobian

$$\dot{\mathbf{p}} = \mathbf{J}_p(q)\ \dot{\mathbf{q}}$$

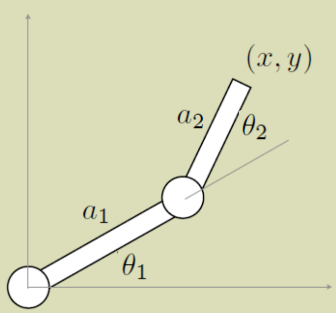
- $\dot{\mathbf{p}}$: endpoint velocity
- $\mathbf{J}_p(q)$: Jacobian matrix
- $\dot{\mathbf{q}}$: joint velocity

For an n-dimensional joint variable space and a cartesian workspace, the Jacobian is a 3xn matrix composed of the partial derivatives of the end-effector position with respect to each joint variable.

$$\mathbf{J}_p = \begin{bmatrix} \frac{\delta x}{\delta q_1} & \frac{\delta x}{\delta q_2} & \cdots & \frac{\delta x}{\delta q_n} \\ \frac{\delta y}{\delta q_1} & \frac{\delta y}{\delta q_2} & \cdots & \frac{\delta y}{\delta q_n} \\ \frac{\delta z}{\delta q_1} & \frac{\delta z}{\delta q_2} & \cdots & \frac{\delta z}{\delta q_n} \end{bmatrix}$$

The Jacobian: Position Jacobian

Example 1: Planar RR



From the forward kinematics, we can extract the position vector from the last column of the transform matrix:

$$\mathbf{d}_2^0 = \begin{bmatrix} a_2c_{12} + a_1c_1 \\ a_2s_{12} + a_1s_1 \\ 0 \end{bmatrix}$$

Taking the partial derivative with respect to each joint variable produces the Jacobian:

The Jacobian: Position Jacobian

Example 1: Planar RR

From the forward kinematics, we can extract the position vector from the last column of the transform matrix:

$$\mathbf{d}_2^0 = \begin{bmatrix} a_2c_{12} + a_1c_1 \\ a_2s_{12} + a_1s_1 \\ 0 \end{bmatrix}$$

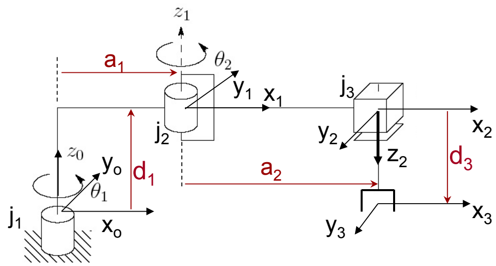
Taking the partial derivative with respect to each joint variable produces the Jacobian:

$$= \begin{bmatrix} -a_1s_1 - a_2s_{12} & -a_2s_{12} \\ a_1c_1 + a_2c_{12} & a_2c_{12} \\ 0 & 0 \end{bmatrix}$$

The Jacobian: Position Jacobian

Example 2: SCARA

$$T_3^0 = \begin{bmatrix} c_{12} & s_{12} & 0 & a_1c_1 + a_2c_{12} \\ s_{12} & -c_{12} & 0 & a_1s_1 + a_2s_{12} \\ 0 & 0 & -1 & d_1 - d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



$$x = a_1c_1 + a_2c_{12}$$
$$y = a_1s_1 + a_2s_{12}$$
$$z = d_1 - d_3$$

$$q = \begin{bmatrix} \theta_1 \\ \theta_2 \\ d_3 \end{bmatrix}$$

$$J_p = \begin{bmatrix} \frac{\partial x}{\partial \theta_1} & \frac{\partial x}{\partial \theta_2} & \frac{\partial x}{\partial d_3} \\ \frac{\partial y}{\partial \theta_1} & \frac{\partial y}{\partial \theta_2} & \frac{\partial y}{\partial d_3} \\ \frac{\partial z}{\partial \theta_1} & \frac{\partial z}{\partial \theta_2} & \frac{\partial z}{\partial d_3} \end{bmatrix} = \begin{bmatrix} -a_1s_1 - a_2s_{12} & -a_2s_{12} & 0 \\ a_1c_1 + a_2c_{12} & a_2c_{12} & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\mathbf{J}_p = \begin{bmatrix} \frac{\delta x}{\delta q_1} & \frac{\delta x}{\delta q_2} & \cdots & \frac{\delta x}{\delta q_n} \\ \frac{\delta y}{\delta q_1} & \frac{\delta y}{\delta q_2} & \cdots & \frac{\delta y}{\delta q_n} \\ \frac{\delta z}{\delta q_1} & \frac{\delta z}{\delta q_2} & \cdots & \frac{\delta z}{\delta q_n} \end{bmatrix}$$

The Jacobian: Singularities

Singularities are points in the configuration space where infinitesimal motion in a certain direction is not possible and the manipulator loses one or more degrees of freedom

when operating at a singular point, bounded end-effector velocities may correspond to unbounded joint velocities

singularities are often found on the extents of the workspace, and also relate to the nonuniqueness of solution to inverse kinematics

Mathematically, singularities exist at any point in the workspace where the Jacobian matrix loses rank.

[i.e. all columns of J are not linearly independent]

The Jacobian: Identifying Singularities

a matrix is singular if and only if it's determinant is zero:

$$\det(\mathbf{J}) = 0$$

The 2×2 matrix,

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

has determinant

$$\det(A) = ad - bc.$$

The 3×3 matrix:

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}.$$

Using the cofactor expansion on the first row of the matrix we get:

$$\det(A) = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$$
$$= aei - afh - bdi + bfg + cdh - ceg$$
$$= (aei + bfg + cdh) - (gec + hfa + idb)$$

[http://en.wikipedia.org/wiki/Determinant]

The Jacobian: Singularities

Example 1: Planar RR

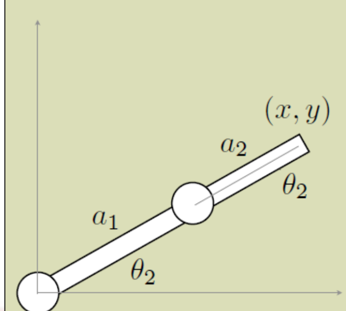
The 2×2 matrix,

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

has determinant

$$\det(A) = ad - bc.$$

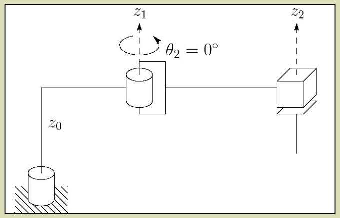
$$\mathbf{J} = \begin{bmatrix} -a_1s_1 - a_2s_{12} & -a_2s_{12} \\ a_1c_1 + a_2c_{12} & a_2c_{12} \end{bmatrix}$$



The Jacobian: Singularities

Example 2: SCARA

$$J = \begin{bmatrix} -a_1 s_1 - a_2 s_{12} & -a_2 s_{12} & 0 \\ a_1 c_1 + a_2 c_{12} & a_2 c_{12} & 0 \\ 0 & 0 & -1 \end{bmatrix}$$



The 3×3 matrix:

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}.$$

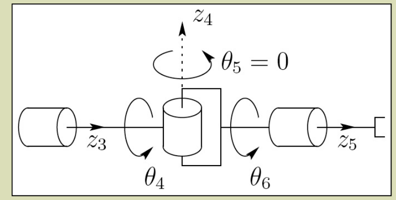
Using the cofactor expansion on the first row of the matrix we get:

$$\det(A) = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$$
$$= aei - afh - bdi + bfg + cdh - ceg$$
$$= (aei + bfg + cdh) - (gec + hfa + idb)$$

The Jacobian: Decoupling of Singularities

Manipulator Singularities

= Wrist Singularities + Arm Singularities



θ_5 = 0 or π

(z_3 and z_5 are collinear)

Compute Jacobian using o_c instead of o_n

The Jacobian: Jacobian Transpose

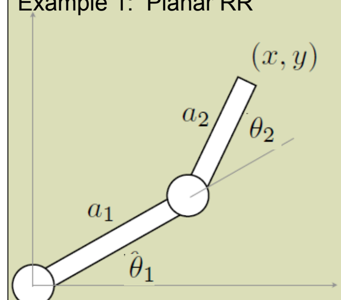
The transpose of the Jacobian relates joint torques and forces to cartesian end-effector forces

$$\tau = \mathbf{J}^\top(q)\,\mathbf{F}$$

- τ: joint torques
- $\mathbf{J}^\top(q)$: Jacobian matrix transpose
- $\mathbf{F}$: endpoint forces

The Jacobian: Jacobian Transpose

Example 1: Planar RR



Beginning the with the standard Jacobian

$$\mathbf{J} = \begin{bmatrix} -a_1 s_1 - a_2 s_{12} & -a_2 s_{12} \\ a_1 c_1 + a_2 c_{12} & a_2 c_{12} \end{bmatrix}$$

We can solve for the joint torques necessary to exert a desired force at the end effector using the Jacobian transpose

$$\tau = \mathbf{J}^\top(q)\,\mathbf{F}$$

The Jacobian: Inverse Jacobian

The Jacobian relationship:

$$\dot{\mathbf{p}} = \mathbf{J}_p(q)\ \dot{\mathbf{q}}$$

Specifies the end-effector velocity that will result when the joints move with velocity $\dot{\mathbf{q}}$

Inverse problem: Find the joint velocities $\dot{\mathbf{q}}$ that produce the desired end-effector velocity

$$\dot{\mathbf{q}} = \mathbf{J}_p(q)^{-1}\ \dot{\mathbf{p}}$$

[Hard if have non-square J → pseudo-inverse (pinv)]

Announcements

Announcements: Term Project Theme- Robotic Art Installation

- Various artistic robotic assignments throughout the course term
  - Labs
  - Midterm Project
  - Multiple specific events towards end of the semester
- Mixed media, sculpture, dance, etc.
  - Kinematics, path planning
  - Localization, image processing
  - Coordination

Announcements

- Homework # 3
- Reading
  - Spong Ch. 4 (today's lecture)
  - Spong Ch. 5 (next lecture)
- Lab 1
  - Art Installation Logo
  - Kinematic robot arm,
  - 2 setups in EAS 001
    - Teams must take turns
    - Many preliminary steps of lab may be done concurrently by different teams (do not require operating computer/robot)