ME 598: Introduction to Robotics

Lecture 1: Course Overview
Robotics Introduction
Rotations and Transformations

Stevens Institute of Technology
Dr. Mishah U. Salman
Fall 2013

Date:
By:

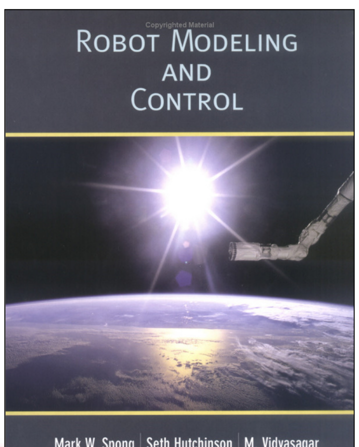
STEVENS INSTITUTE of TECHNOLOGY THE INNOVATION UNIVERSITY

Slides adapted from Dr. David J. Cappelleri

Course Overview: Course Description

- Introduction to fundamental concepts of robotic manipulators and mobile robots
- Topics to be covered include:
  - Forward and inverse kinematics
  - DH parameters
  - The Jacobian
  - Trajectory planning
  - Feedback control
  - Actuators and sensors
  - Mobile robot kinematics
  - Computer vision
  - Localization
  - Motion planning
- Hands-on lab and project assignments using Intelitek SCOREBOT robotic manipulator and iRobot Create mobile robot platform

Course Overview: Textbooks



Course Overview: Robots and Accessories

- Intelitek Scorebot 4PC

- TRENDNet Wireless Internet Camera

- iRobot Create

- Element Direct BAM (Bluetooth Adapter Module) and Bluetooth Dongle

Course Overview: Tentative Schedule

Term Week	Material
1	Course Overview, Robotics Introduction Rotations and Transformations
2	NO CLASS
3	Forward Kinematics, DH Parameters, Inverse Kinematics
4	Velocity Kinematics- the Jacobian Midterm Project Intro
5	Path and Trajectory Planning
6	Independent Joint Control
7	Actuators and Sensors
8	MIDTERM EXAM (in-class)
9	Midterm Solution Review Final Project Intro Midterm Project Demos
10	Mobile Robot Intro/Kinematics Mobile Robot Lab 1
11	Computer Vision/Image Processing, Sensor-Based Navigation Mobile Robot Lab 2
12	Localization, Path Planning and Navigation Mobile Robot Lab 3
13	*Final Project Lab Session - I
14	NO CLASS (Thanksgiving Break)
15	*Final Project Lab Session - II
16	*Final Project Lab Session - III
17	Final Report Due

Course Overview: ME 598 Staff

Instructor:

Mishah U. Salman, Ph.D.
Teaching Assistant Professor
Department of Mechanical Engineering
Carnegie Building, Room C-209
Mishah.Salman@stevens.edu
Office hours: Wed. 3:30pm - 5:30pm

Teaching Assistant:

Zhenbo Fu
zfu@stevens.edu
Lab hours: During lab class

Course Overview: Course Logistics

Class website

- https://sites.google.com/site/me598fa2013/
  - Homework Assignments
  - Lab Assignments
  - Lecture Slides
  - Project Materials
  - **Extra lab time signup**
  - Etc.

Course Overview: Class Policies

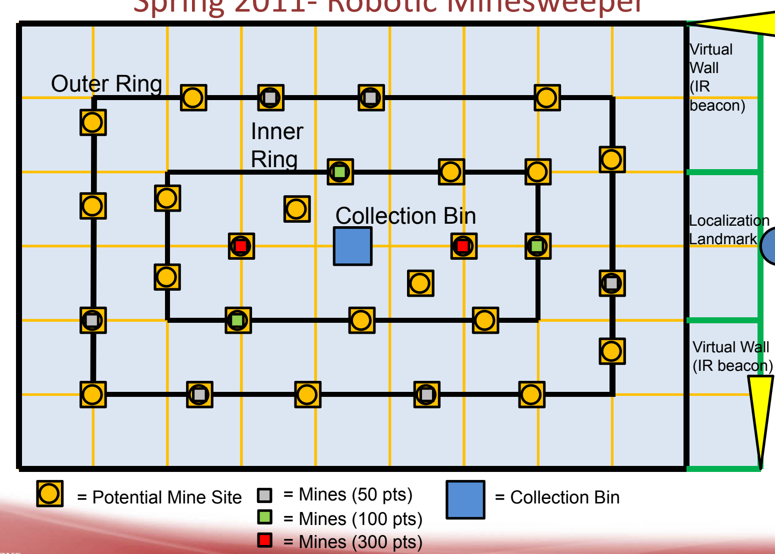
- Homework
  - Periodic
  - Related to the lectures, labs, final project, etc.
    - Problem sets should be submitted individually (not team)
- Laboratory and Project Groups
  - Lab and project exercises done in groups of 3-4 students
  - Students may choose their own group members
  - At the end of the course, each group member will grade the other members based on their performance throughout the semester
    - These evaluations will be factored into the final course grades for each student

Course Overview: Assignments and Evaluation

- Homework Problem Sets: 20%
- Manipulation Lab(s)
- Mobile Robot Labs
- Midterm Exam 20%
- Midterm Project 10%
- Final Project 20%

(Manipulation Lab(s) and Mobile Robot Labs: 30%)

Course Overview: Spring 2011- Robotic Minesweeper



Course Overview: ME 598 Final Project Spring 2012

- Robotic Decathlon
  - 10 Events throughout the course of semester
    - 5 events in the context of labs
    - 2 events as part of midterm project
    - 3 specific Final Project Task events
      - Synchronized "Swimming"
      - Robotic Archery
      - Robo Soccer Shootout

Course Overview: Warning!

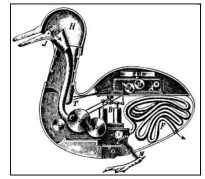
- This class has a heavy lab component
  - It will be a lot of fun
  - You will need to spend ***substantial*** time working in teams on the labs outside of class
    - Labs and projects constitute a significant portion of your final grade

Robotics Introduction: What is a "Robot"?

Robotics Introduction: History

322 B.C. - "If every tool, when ordered, or even of its own accord, could do the work that befits it... then there would be no need either of apprentices for the master workers or of slaves for the lords." - Aristotle

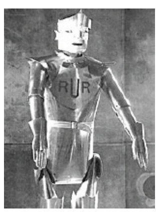
1495 - Leonardo da Vinci designs a mechanical clockwork that sits up, waves its arms, and moves its head



1738 - Jacques de Vaucanson creates a mechanical duck that was able to eat, flap its wings, and excrete

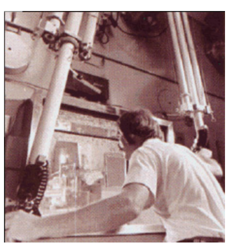
1769 - Wolfgang von Kempelen builds "The Turk", which gains fame as an automaton capable of playing chess - until the hidden human operator was discovered!

Robotics Introduction: History



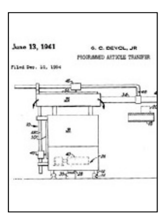
1921 - Karel Capek popularizes the term "robot" in a play called *R.U.R. (Rossum's Universal Robots)* wherein robot workers take over the earth

1942 - Isaac Asimov publishes *Runaround*, which introduces the three "laws" of robotics

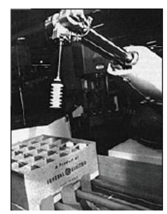


1951 - Raymond Goertz builds the first master/slave teleoperation system for handling radioactive material

1954 - George Devol files a patent for the first programmable robot, and calls it "universal automation"



Robotics Introduction: History



1961- *Unimate*, the first industrial robot, begins work on a General Motors assembly line

Present Day

- "A robot is a reprogrammable, multifunctional manipulator designed to move material, parts, tools, or specialized devices through variable programmed motions for the performance of a variety of tasks."
  - The Robotics Institute of America

Robotics Introduction: Examples

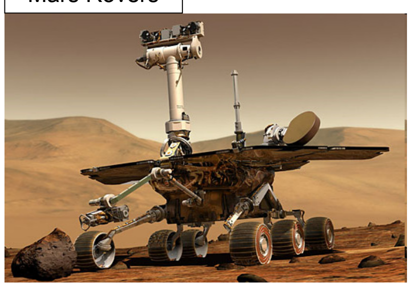
Robotics Introduction: Aren't robots more/different…?

- No single correct definition of "robot", but a typical robot will have several or possibly all of the following properties:
  - It is artificially created
  - It can sense its environment, and manipulate or interact with things in it
  - It has some ability to make choices based on the environment, often using automatic control or a preprogrammed sequence
  - It is programmable
  - It moves with one or more axes of rotation or translation
  - It makes dexterous coordinated movements
  - It moves without direct human intervention
  - It appears to have intent or agency

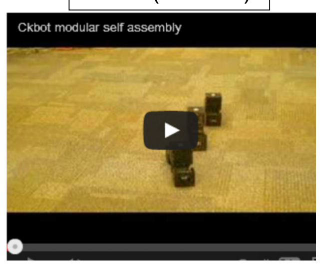
[Wikipedia]

Robotics Introduction: Examples

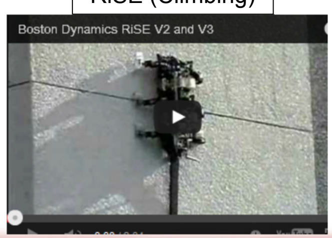
Mars Rovers



ckBot (Modular)



RiSE (Climbing)

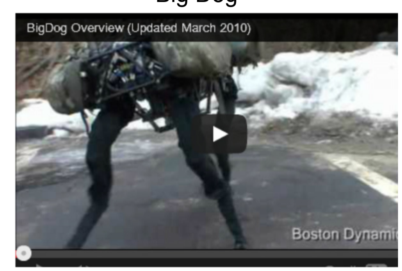


Roomba

Robotics Introduction: Examples- Legged

Little Dog

Big Dog



RHex

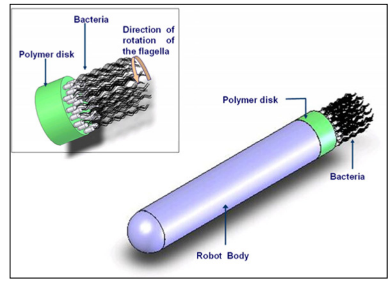
Robotics Introduction: Examples- Medical

Intuitive Surgical: *da Vinci* Surgical System

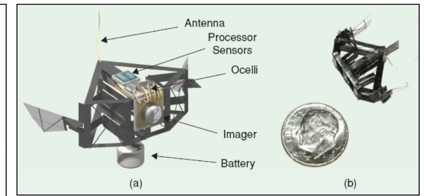
- Patient's Side Cart
  - Holds robotic arms that manipulate instruments
  - Three arms standard: surgeon right, left to hold instruments, endoscope
  - Optional fourth arm to hold another instrument
- Surgeon's Console
  - Surgeon sits here during procedure
  - Ergonomic design
  - Provides 3D image of surgical field
  - Instrument controls below display
- 3-D Vision System
  - 3-D endoscope
  - Progressive scan color monitors
  - State-of-the-art processing equipment



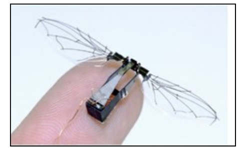

Robotics Introduction: Examples- Micro/Nano Robots



[Behkam and M. Sitti, 2007]

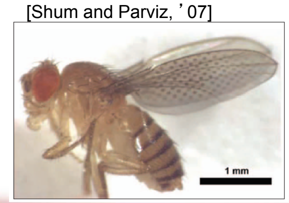


[Micromechanical Flying Insect (MFI) project, Fearing et al., 1998-present]

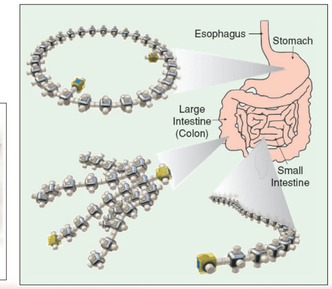


[http://robobees.seas.harvard.edu]

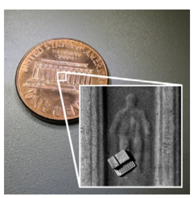
[Shum and Parviz, '07]



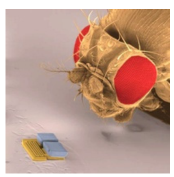
[ARES Project, www.ares-nest.org]



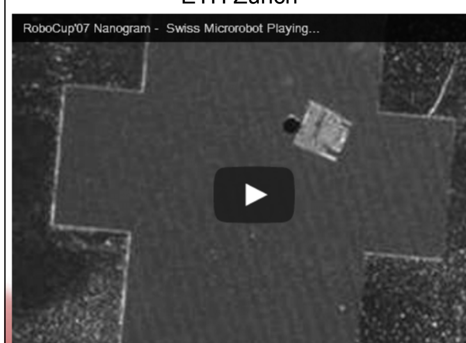
Robotics Introduction: Examples- Micro/Nano Robots



ETH Zurich



Carnegie Mellon University



Robotics Introduction: Example- Humanoid Robots

Robonaut

ASIMO – **A**dvanced **S**tep in **I**nnovative **MO**bility


Robotics Introduction:
Example: Humanoid Robots

PETMAN

Robotics Introduction: Building Robots on-the-fly

Robotics Introduction: Examples: Fully Autonomous
DARPA Grand Challenge Robots

1st place: CMU - Boss

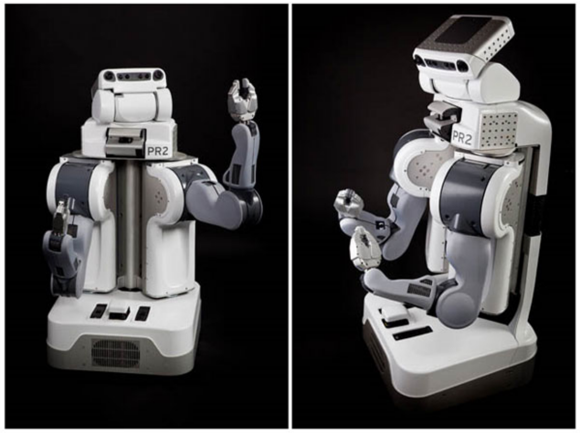
2nd place: Stanford - Stanley

3rd place: VT - Victor Tango

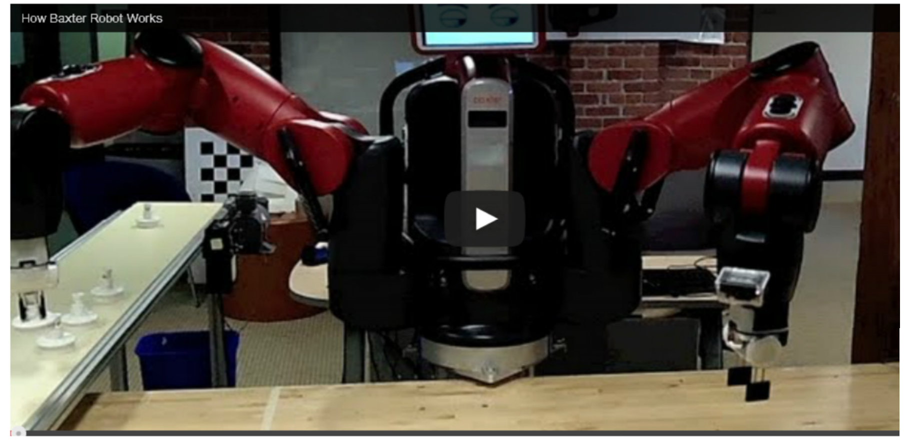
4th place: UPenn/Lehigh - Little Ben

Robotics Introduction: Examples: Fully Autonomous
Kiva Systems

Robotics Introduction: Willow Garage PR2

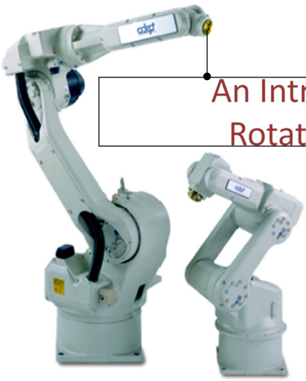


Robotics Introduction: Rethink Robotics- Baxter



Robotics Introduction: Aerial Robots

Robotics Introduction: Where does this class fit in?

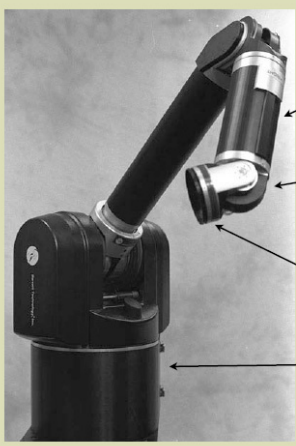


An Introduction to Manipulators: Rotations and Transformations

* Most slides courtesy of Jonathan Fiene, University of Pennsylvania

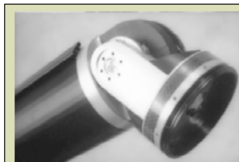
Manipulator Basics

Rotations and Transformations: General Terminology

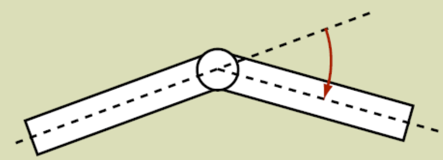


- Link : rigid body, 6 degrees of freedom
- Joint : connection between two links
- End-effector : interacts with the environment
- Base : connected to ground

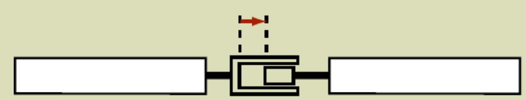
Rotations and Transformations: Joint Descriptions



(R)evolute : angular displacement between adjacent links

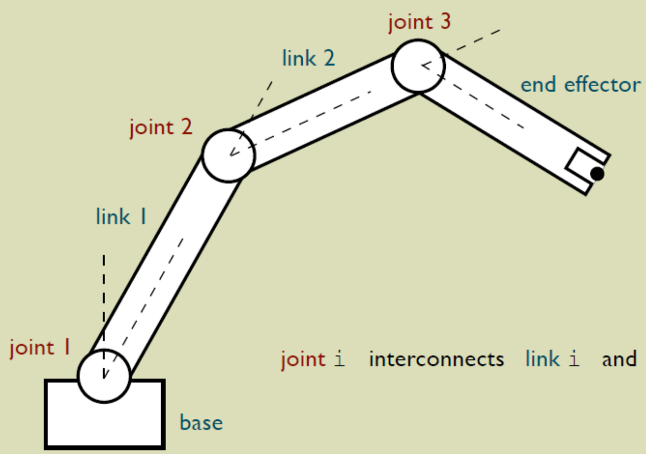


(P)rismatic : linear displacement between adjacent links



Rotations and Transformations: Kinematic Chains

A **kinematic chain** is a system of rigid bodies connected by joints

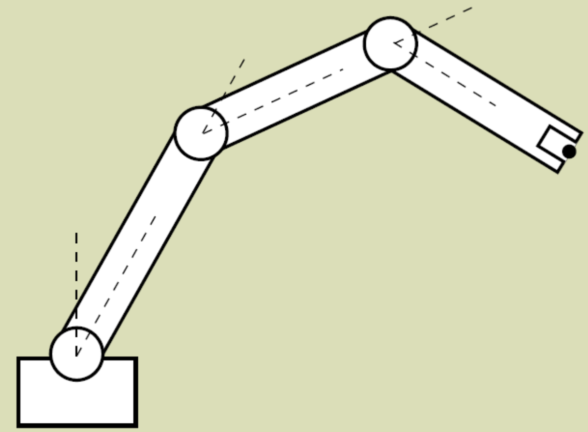


joint i interconnects link i and link i−1

In a **serial** kinematic chain, each intermediate link is connected to two others

Rotations and Transformations: Configuration Space

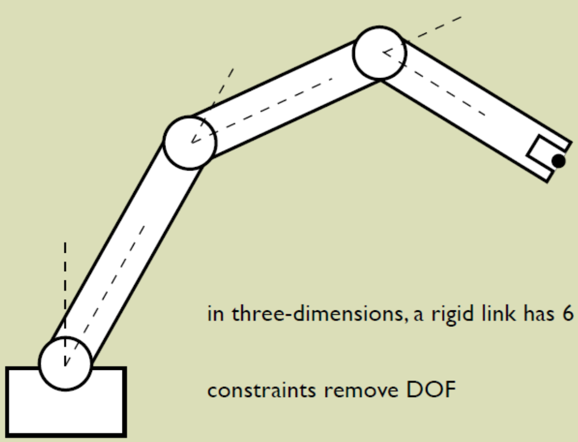
The **configuration** defines the location of every point of the manipulator



The **configuration space** is the set of all possible configurations

Rotations and Transformations: Degrees of Freedom

The **degrees of freedom (DOF)** are the minimum number of parameters necessary to specify the configuration



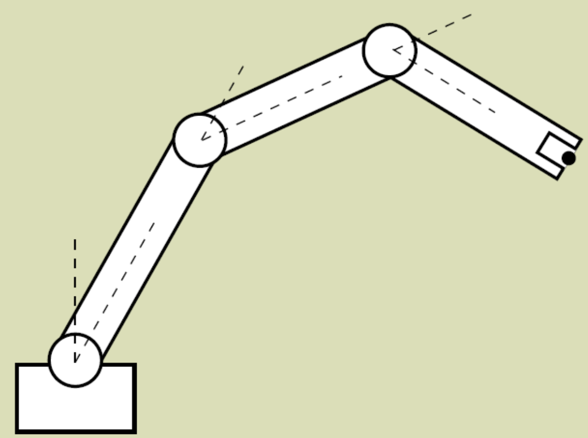
in three-dimensions, a rigid link has 6 DOF

constraints remove DOF

revolute and prismatic joints impose 5 constraints

Rotations and Transformations: Workspace

The **reachable workspace** is the set of points reachable by the end effector

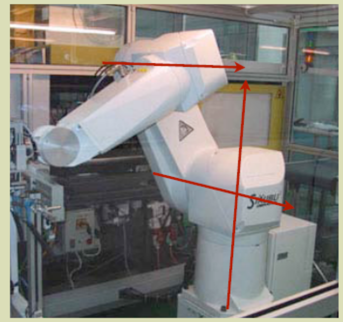


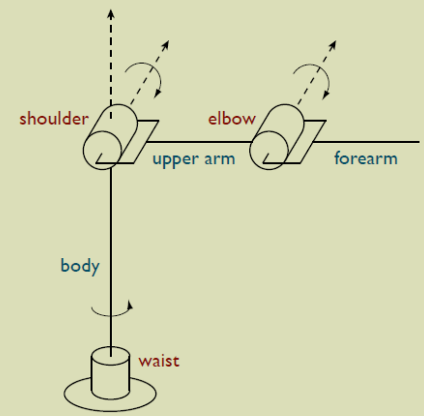
The **dexterous workspace** is a subset of the reachable workspace wherein the end effector can obtain an arbitrary orientation

Common Configurations

Rotations and Transformations: Common Configurations

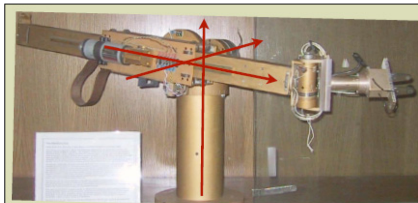
Articulated Manipulator (RRR)





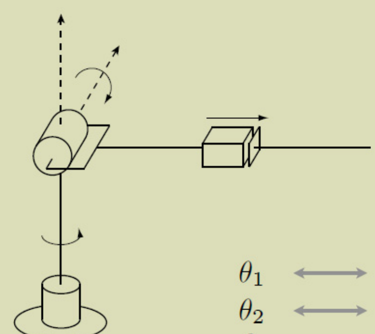
Rotations and Transformations: Common Configurations

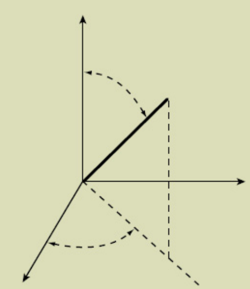
Rotations and Transformations: Common Configurations



Spherical Manipulator (RRP)

joint coordinates map to standard spherical coordinates





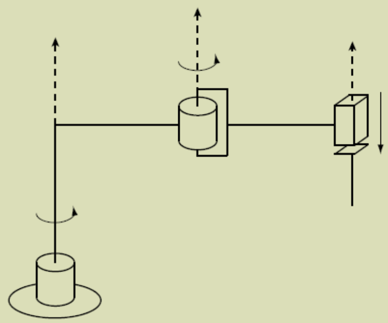
$\theta_1 \longleftrightarrow \phi$

$\theta_2 \longleftrightarrow \theta$

$d_3 \longleftrightarrow r$

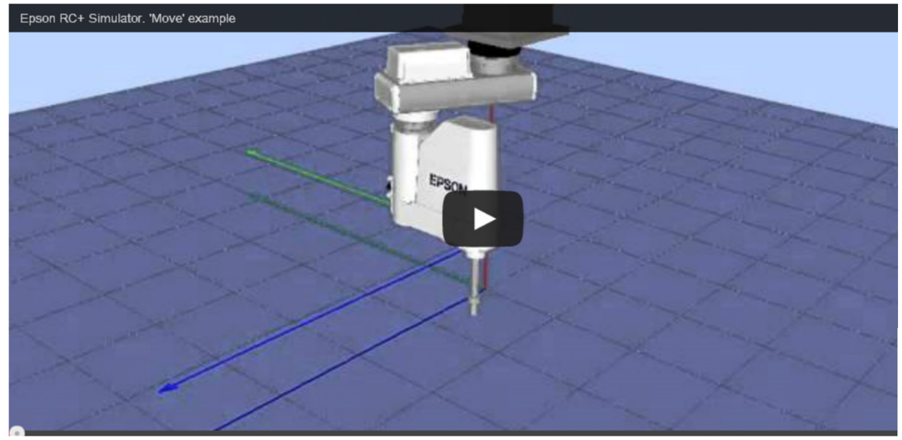
Rotations and Transformations: Common Configurations

Selective Compliant Articulated Robot for Assembly (SCARA)
(RRP)



Introduced in 1979, The SCARA manipulator design revolutionized the assembly of small electronics

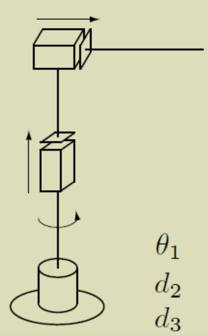
Rotations and Transformations: Common Configurations- SCARA Example



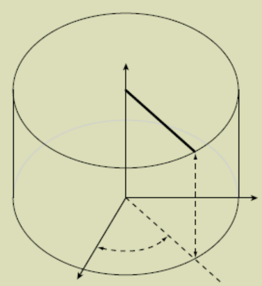
Rotations and Transformations: Common Configurations

Cylindrical Manipulator
(RPP)

joint coordinates map to cylindrical coordinates

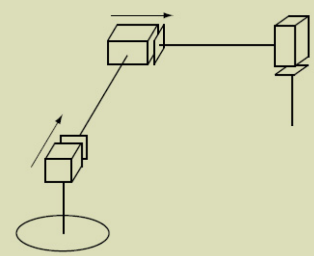


$\theta_1 \longleftrightarrow \theta$
$d_2 \longleftrightarrow z$
$d_3 \longleftrightarrow r$



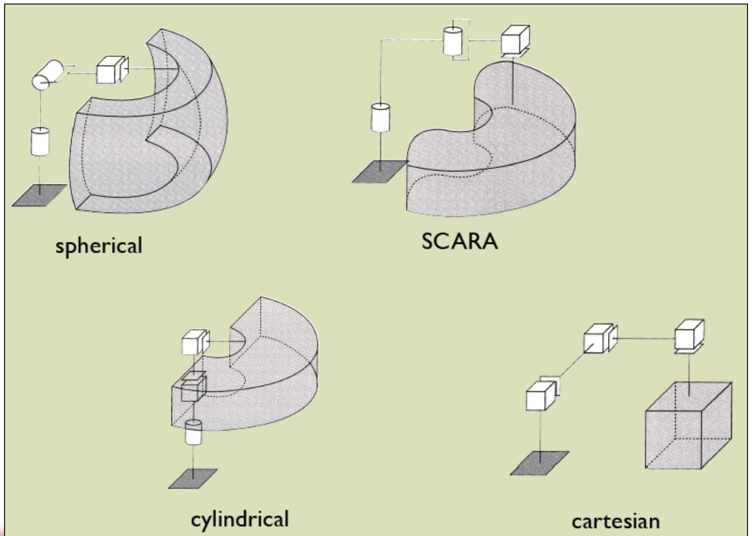
Rotations and Transformations: Common Configurations

Cartesian Manipulator
(PPP)



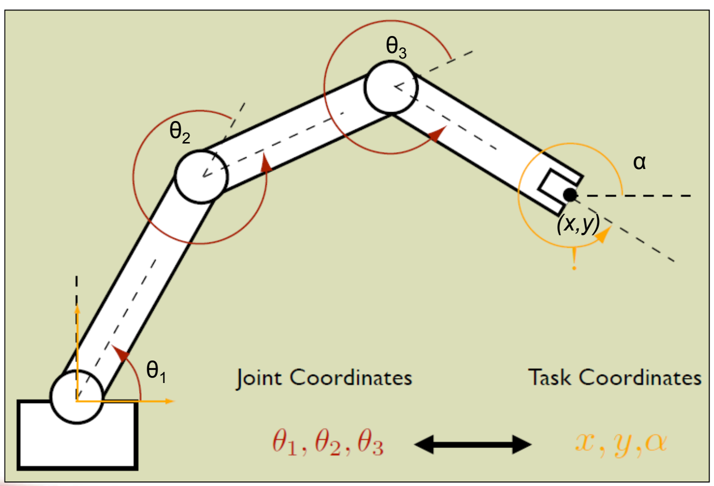
Joint variables directly correspond to the cartesian coordinates of the end-effector

Rotations and Transformations: Reachable Workspaces

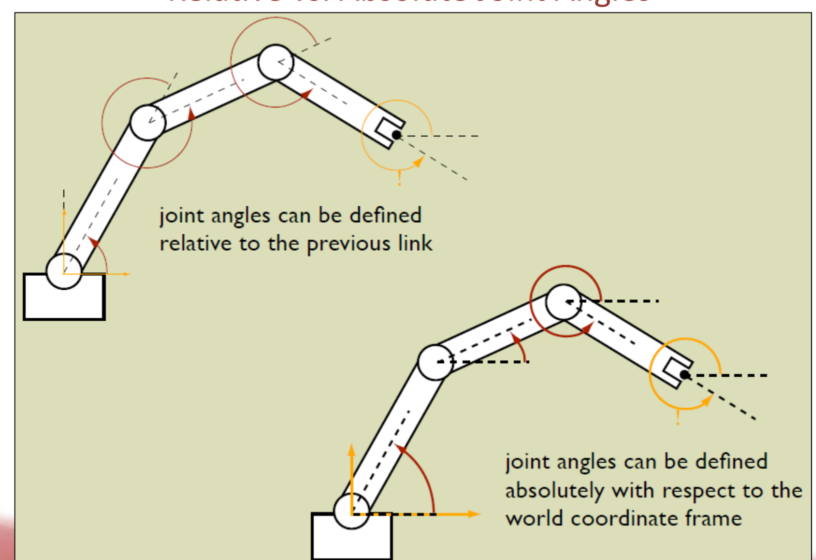


Manipulator Kinematics

Rotations and Transformations: Coordinates

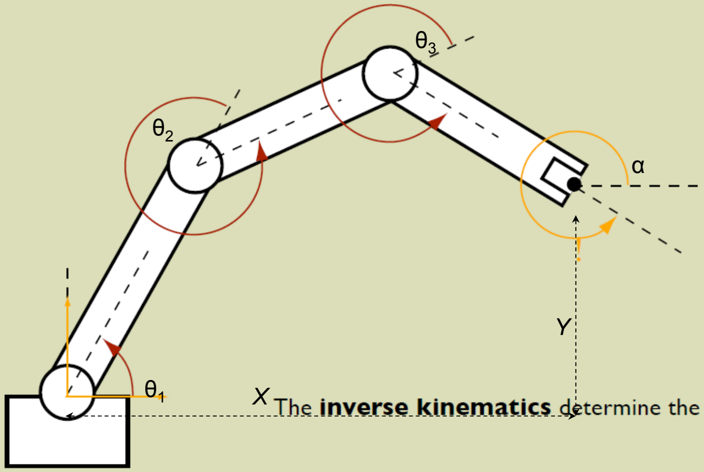


Rotations and Transformations: Relative vs. Absolute Joint Angles



Rotations and Transformations: Position Kinematics

The **forward kinematics** define the end-effector position for a given set of joint values



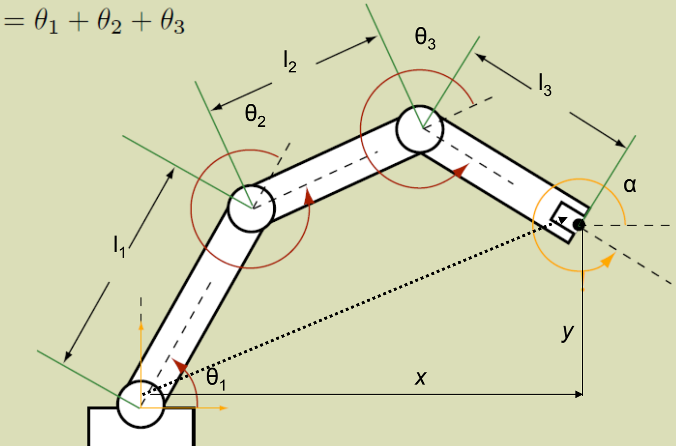
The **inverse kinematics** determine the

Rotations and Transformations: Forward Kinematics via Trigonometry

$$x = l_1 \cos\theta_1 + l_2 \cos(\theta_1 + \theta_2) + l_3 \cos(\theta_1 + \theta_2 + \theta_3)$$

$$y = l_1 \sin\theta_1 + l_2 \sin(\theta_1 + \theta_2) + l_3 \sin(\theta_1 + \theta_2 + \theta_3)$$

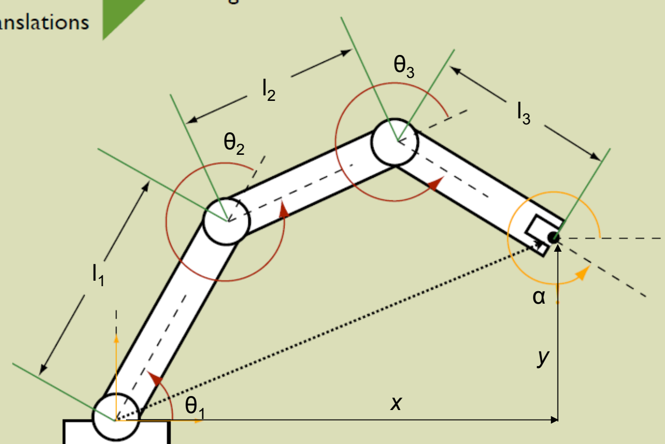
$$\alpha = \theta_1 + \theta_2 + \theta_3$$



Rotations and Transformations: Forward Kinematics

We'll use linear matrices to generalize the solution process

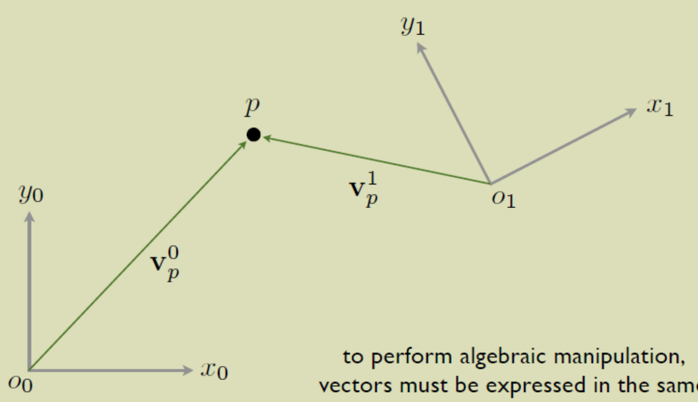
Rotations, Translations ▶ Homogeneous Transformations



Rotations

Rotations and Transformations: Frame Notation

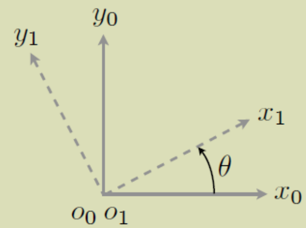
The relative frame is designated using **superscript** notation



to perform algebraic manipulation, vectors must be expressed in the same (or parallel) coordinate frame

$$\mathbf{v}_p^1 + \mathbf{v}_p^2 = \textbf{UNDEFINED}$$

Rotations and Transformations: Planar Coordinate Rotations



project frame 1 into from 0

$$\mathbf{x}_1^0 = \begin{bmatrix} x_1 \cdot x_0 \\ x_1 \cdot y_0 \end{bmatrix} = \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$$

$$\mathbf{y}_1^0 = \begin{bmatrix} y_1 \cdot x_0 \\ y_1 \cdot y_0 \end{bmatrix} = \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix}$$

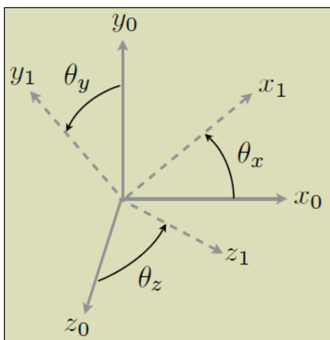
which can be expressed as a **rotation matrix**

$$\mathbf{R}_1^0 = \begin{bmatrix} \mathbf{x}_1^0 & \mathbf{y}_1^0 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

the inverse of which is simply the matrix transpose

$$\mathbf{R}_0^1 = (\mathbf{R}_1^0)^\top = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

Rotations and Transformations: Three-Dimensional Coordinate Rotations



$$\mathbf{R}_1^0 = \begin{bmatrix} x_1 \cdot x_0 & y_1 \cdot x_0 & z_1 \cdot x_0 \\ x_1 \cdot y_0 & y_1 \cdot y_0 & z_1 \cdot y_0 \\ x_1 \cdot z_0 & y_1 \cdot z_0 & z_1 \cdot z_0 \end{bmatrix}$$

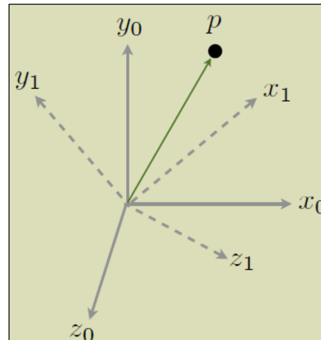
$$\mathbf{R}_{x,\theta} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

$$\mathbf{R}_{y,\theta} = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$\mathbf{R}_{z,\theta} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The **basic rotation matrices** define rotations about the three coordinate axes

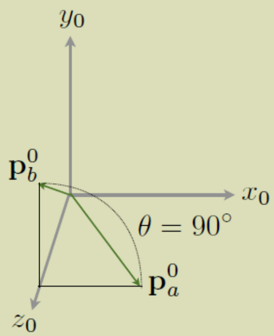
Rotations and Transformations: Rotational Transformations



For pure coordinate rotation, a point in frame 1 can be expressed in frame 0 using the rotation matrix

$$\mathbf{v}_p^0 = \mathbf{R}_1^0\, \mathbf{v}_p^1$$

Rotations and Transformations: Rotational Transformations



The rotation matrix can be used to perform arbitrary rotations on vectors

$$\mathbf{v}_p^0 = \mathbf{R}_1^0 \, \mathbf{v}_p^1$$

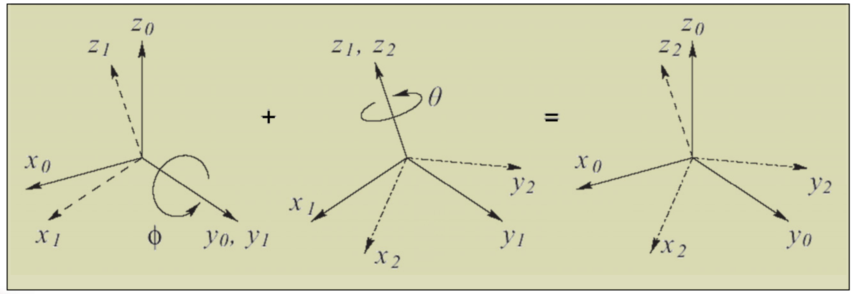
$$\mathbf{p}_a^0 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$\mathbf{p}_b^0 = \mathbf{R}_{z,\theta} \, \mathbf{p}_a^0 = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

Rotations and Transformations: Composition of Rotations (Intermediate Reference)

the result of a successive rotation about an intermediate frame can be found by **post-multiplying** by the corresponding rotation matrix

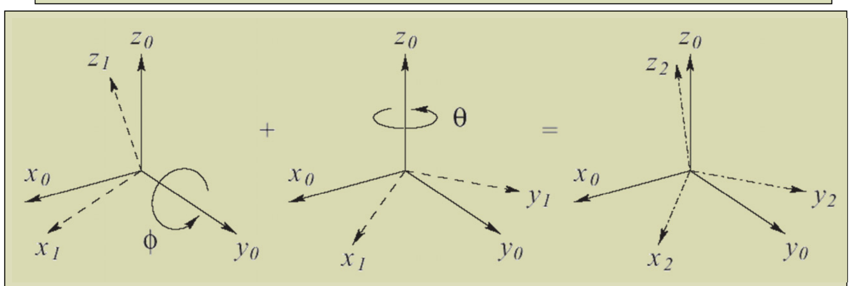
$$\mathbf{R}_2^0 = \mathbf{R}_1^0 \, \mathbf{R}_2^1$$ (using current axes)



Rotations and Transformations: Composition of Rotations (Fixed Reference)

the result of a successive rotation about a fixed frame can be found by **pre-multiplying** by the corresponding rotation matrix

$$\mathbf{R}_2^0 = \mathbf{R} \, \mathbf{R}_1^0$$



Note that $\mathbf{R}$ is a rotation about the original frame

Rotations and Transformations: Example- Composition of Rotations

- Suppose R is defined by the following sequence of basic rotations in this order:
  - Rotation of θ about current x-axis
  - Rotation of Φ about current z-axis
  - Rotation of α about fixed z-axis
  - Rotation of β about current y-axis
  - Rotation of δ about fixed x-axis

$$R = R_{x,\delta} \; R_{z,\alpha} \; R_{x,\theta} \; R_{z,\Phi} \; R_{y,\beta}$$

Rotations and Transformations: Parameterization of Rotations

in three-dimensions, no more than 3 values are needed to specify an arbitrary rotation

$$\mathbf{R}_1^0 = \begin{bmatrix} x_1 \cdot x_0 & y_1 \cdot x_0 & z_1 \cdot x_0 \\ x_1 \cdot y_0 & y_1 \cdot y_0 & z_1 \cdot y_0 \\ x_1 \cdot z_0 & y_1 \cdot z_0 & z_1 \cdot z_0 \end{bmatrix}$$

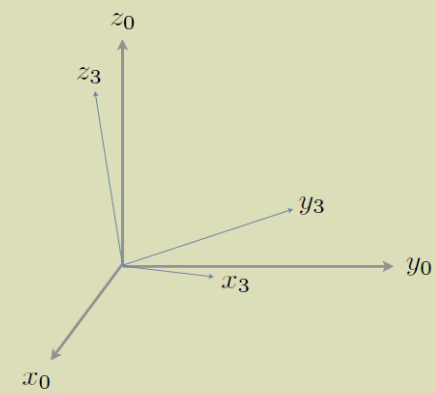
the 9-element rotation matrix has at least 6 redundancies

numerous methods have been developed to represent rotation/orientation with only 3 variables

Euler Angles   Roll, Pitch, Yaw Angles   Axis/Angle Representation

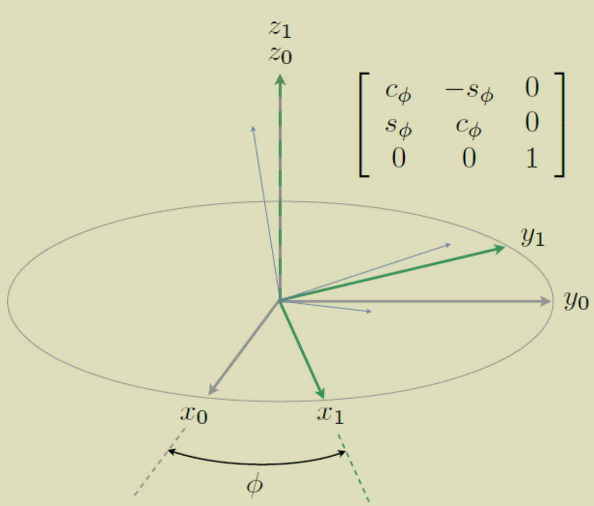
Rotations and Transformations: Euler Angles

Define a set of three **intermediate** angles, ϕ, θ, ψ , to go from $0 \longrightarrow 3$



Rotations and Transformations: Euler Angles

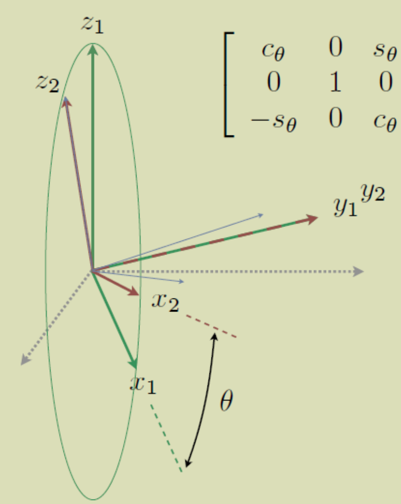
step 1: rotate by ϕ about z_0



$$\begin{bmatrix} c_\phi & -s_\phi & 0 \\ s_\phi & c_\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Rotations and Transformations: Euler Angles

step 2: rotate by θ about y_1

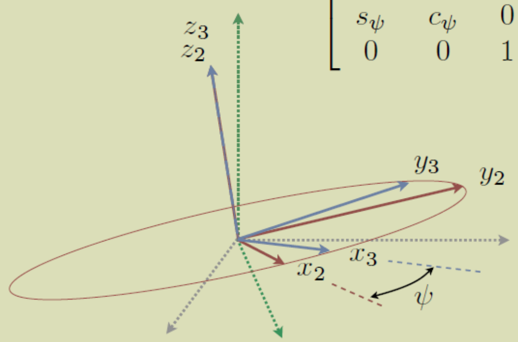


$$\begin{bmatrix} c_\theta & 0 & s_\theta \\ 0 & 1 & 0 \\ -s_\theta & 0 & c_\theta \end{bmatrix}$$

Rotations and Transformations: Euler Angles

step 3: rotate by ψ about z_2

$$\begin{bmatrix} c_\psi & -s_\psi & 0 \\ s_\psi & c_\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



Rotations and Transformations: Euler Angles to Rotation Matrices

(**post**-multiply using the **basic rotation matrices**)

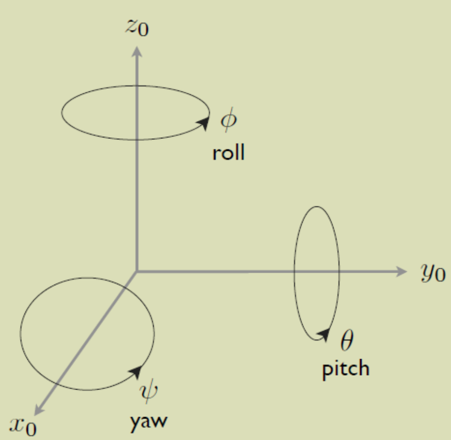
$$\mathbf{R} = \mathbf{R}_{z,\phi}\,\mathbf{R}_{y,\theta}\,\mathbf{R}_{z,\psi}$$

$$= \begin{bmatrix} c_\phi & -s_\phi & 0 \\ s_\phi & c_\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_\theta & 0 & s_\theta \\ 0 & 1 & 0 \\ -s_\theta & 0 & c_\theta \end{bmatrix} \begin{bmatrix} c_\psi & -s_\psi & 0 \\ s_\psi & c_\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c_\phi c_\theta c_\psi - s_\phi s_\psi & -c_\phi c_\theta s_\psi - s_\phi c_\psi & c_\phi s_\theta \\ s_\phi c_\theta c_\psi + c_\phi s_\psi & -s_\phi c_\theta s_\psi + c_\phi c_\psi & s_\phi s_\theta \\ -s_\theta c_\psi & s_\theta s_\psi & c_\theta \end{bmatrix}$$

Rotations and Transformations: Roll, Pitch, Yaw Angles

defined as a set of three angles about a **fixed** reference



Rotations and Transformations: Roll, Pitch, Yaw Angles to Rotation Matrices

(**pre**-multiply using the **basic rotation matrices**)

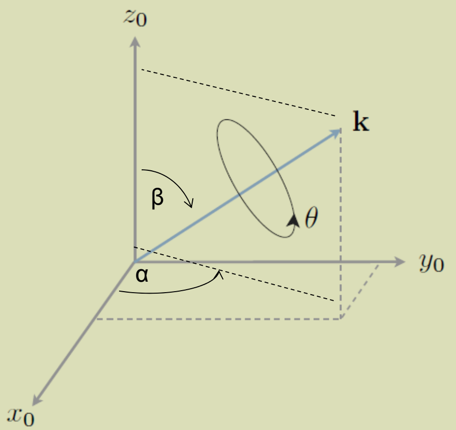
$$\mathbf{R} = \mathbf{R}_{z,\phi}\,\mathbf{R}_{y,\theta}\,\mathbf{R}_{x,\psi}$$

$$= \begin{bmatrix} c_\phi & -s_\phi & 0 \\ s_\phi & c_\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_\theta & 0 & s_\theta \\ 0 & 1 & 0 \\ -s_\theta & 0 & c_\theta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_\psi & -s_\psi \\ 0 & s_\psi & c_\psi \end{bmatrix}$$

$$= \begin{bmatrix} c_\phi c_\theta & c_\phi s_\theta s_\psi - s_\phi c_\psi & s_\phi s_\psi + c_\phi s_\theta c_\psi \\ s_\phi c_\theta & s_\phi s_\theta s_\psi + c_\phi c_\psi & s_\phi s_\theta c_\psi - c_\phi s_\psi \\ -s_\theta & c_\theta s_\psi & c_\theta c_\psi \end{bmatrix}$$

Rotations and Transformations: Axis/Angle Representation

rotation by an angle about an arbitrary axis in space



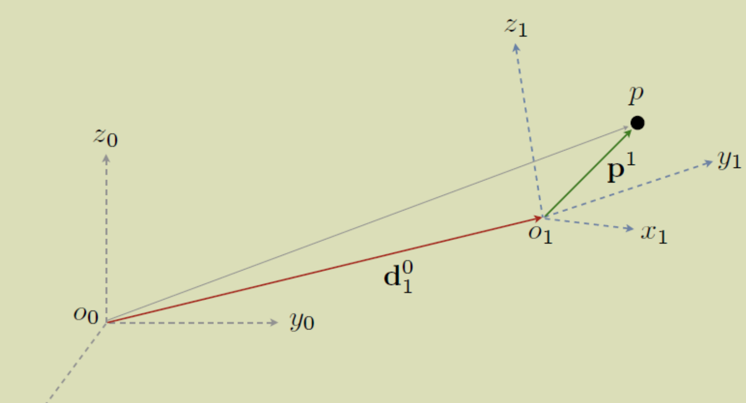
$$\mathbf{k} = \begin{bmatrix} k_x \\ k_y \\ k_z \end{bmatrix}$$

{see pages 57-59 for the transformations to/from rotation matrices}

Homogeneous Transformations

Rotations and Transformations: Rigid Motions

a **rigid motion** couples pure translation with pure rotation



rigid motions can be expressed as

$$\mathbf{p}^0 = \mathbf{R}_1^0\,\mathbf{p}^1 + \mathbf{d}_1^0$$

Rotations and Transformations: Homogeneous Transformations

a **homogeneous transform** is a matrix representation of rigid motion, defined as

$$\mathbf{H} = \begin{bmatrix} \mathbf{R} & \mathbf{d} \\ \mathbf{0} & 1 \end{bmatrix}$$

where $\mathbf{R}$ is the 3x3 rotation matrix, and $\mathbf{d}$ is the 1x3 translation vector

$$\mathbf{H} = \begin{bmatrix} n_x & s_x & a_x & d_x \\ n_y & s_y & a_y & d_y \\ n_z & s_z & a_z & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

the **inverse** of a homogeneous transform can be expressed as

$$\mathbf{H}^{-1} = \begin{bmatrix} \mathbf{R}^\top & -\mathbf{R}^\top d \\ 0 & 1 \end{bmatrix}$$

Rotations and Transformations: Homogeneous Transformations

the **homogeneous representation** of a vector is formed by concatenating the original vector with a unit scalar

$$\mathbf{P} = \begin{bmatrix} \mathbf{p} \\ 1 \end{bmatrix}$$

where $\mathbf{p}$ is the 1x3 vector

$$\mathbf{P} = \begin{bmatrix} p_x \\ p_y \\ p_z \\ 1 \end{bmatrix}$$

Rotations and Transformations: Homogeneous Transformations

rigid body transformations are accomplished by pre-multiplying by the homogenous transform

$$\mathbf{P}^0 = \mathbf{H}_1^0 \, \mathbf{P}^1$$

composition of multiple transforms is the same as for rotation matrices:

post-multiply when successive rotations are relative to intermediate frames

$$\mathbf{H}_2^0 = \mathbf{H}_1^0 \, \mathbf{H}_2^1$$

pre-multiply when successive rotations are relative to the first fixed frame

$$\mathbf{H}_2^0 = \mathbf{H} \, \mathbf{H}_1^0$$

Rotations and Transformations: Homogeneous Transformations

$$Trans_{x,a} = \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad Rot_{x,\alpha} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_\alpha & -s_\alpha & 0 \\ 0 & s_\alpha & c_\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Trans_{y,b} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad Rot_{y,\beta} = \begin{bmatrix} c_\beta & 0 & s_\beta & 0 \\ 0 & 1 & 0 & 0 \\ -s_\beta & 0 & c_\beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Trans_{z,c} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad Rot_{z,\gamma} = \begin{bmatrix} c_\gamma & -s_\gamma & 0 & 0 \\ s_\gamma & c_\gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotations and Transformations: Example- Homogeneous Transformations

- Find H that represents the following in order:
  - Rotation by angle α about current x-axis
  - Translation of b units along current x-axis
  - Translation of d units along current z-axis
  - Rotation by angle θ about current z-axis

H = $Rot_{x,\,\alpha}$ $Trans_{x,\,\beta}$ $Trans_{z,\delta}$ $Rot_{z,\,\theta}$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_\alpha & -s_\alpha & 0 \\ 0 & s_\alpha & c_\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & b \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} c_\theta & -s_\theta & 0 & 0 \\ s_\theta & c_\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H = \begin{bmatrix} c_\theta & -s_\theta & 0 & b \\ c_\alpha s_\theta & c_\alpha & -s_\alpha & -ds_\theta \\ s_\alpha s_\theta & s_\alpha & c_\alpha & dc_\alpha \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotations and Transformations: 5-axis CNC Machining

Announcements & Assignments

- No class on week 2 (09/04)
  - Classes resume week 3 (09/11)
- Reading: Spong Ch. 1 and 2
- Homework 1
  - Due 09/11 @ the beginning of class
  - **To be posted on course website**
- Reading for next class: Spong Ch. 3